This question paper contains 4 printed pages. 4/12/17

Your Roll No. ....

Sl. No. of Ques. Paper: 5712 (1) **H**

Unique Paper Code : 235501

Name of Paper : **Differential Equations – III**

Name of Course : **B.Sc. (Hons.) Mathematics**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

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(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
Attempt any three parts from each question.

1. (a) For $0 < a < b$, let the function $f(t)$ be defined as:

$$f(t) = \begin{cases} 1 & \text{if } a \le t < b, \\ 0 & \text{if } t < a \text{ or } t \ge b. \end{cases}$$

Express $f(t)$ in terms of the unit step function to show that

$L(f(t)) = s^{-1}(e^{-as} - e^{-bs})$. **(6)**

(b) (i) Solve the initial value problem using Laplace transform :

$x''(t) + 4x'(t) + 8x(t) = e^{-t}, x(0) = 0, x'(0) = 0.$ **(4)**

(ii) Find the inverse Laplace transform of $F(s) = \dfrac{2s+1}{s(s^2+9)}$. **(2)**

(c) Define a regular singular point of the homogenous second-order linear differential equation

$A(x)y'' + B(x)y' + C(x)y = 0.$

Investigate the nature of the point $x = 0$ for the differential equation

$x^4y'' + (x^2 \sin x)y' + (1 - \cos x)y = 0.$ **(6)**

(d) Find a Frobenius series solution of Bessel's Equation of order zero

P. T. O.

(6)

$$x^2y'' + xy' + x^2y = 0.$$

2. (a) (i) Use linear congruence method to generate 10 random numbers using $a=1, b=7, c=9$ and $x_0 = 27$. Was there cycling? If so, when did it occur? (3)

(ii) Use middle square method to generate a sequence of random numbers using $x_0 = 1003$ till it degenerates to zero. (3)

(b) Using Monte Carlo simulation, write an algorithm to calculate an approximation to π by considering the number of random points selected inside the quarter circle $Q: x^2 + y^2 = 1$, for $x \geq 0,\ y \geq 0$ where the quarter circle is taken to be inside the square $S: 0 \leq x \leq 1$ and $0 \leq y \leq 1$. Use the equation that $\pi/4$=area Q/area S. (6)

(c) Fit the model to the data using Chebyshev's criterion to minimize the largest deviation.

$$y = cx$$

y	11	25	54	90
x	5	10	20	30

Hence, solve the optimization problem graphically that determines the parameter c to minimize to largest absolute deviation. (6)

(d) A harbor has unloading facilities for ships. There is a facility of unloading of only one ship at a time in the harbor. The unloading time required for a ship depends on the type and amount of the cargo and varies from 60 to 120 minutes. The data for 6 ships are given as:

Time (in minutes)	Ship1	Ship2	Ship3	Ship4	Ship5	Ship6
Time between successive ships	30	45	30	140	120	80
Unloading time	60	90	60	120	110	90

(i) Draw the time-line diagram depicting clearly the situation for each ship, the ideal time for the harbor.

(ii) What is the average and maximum time per ship in the harbor?

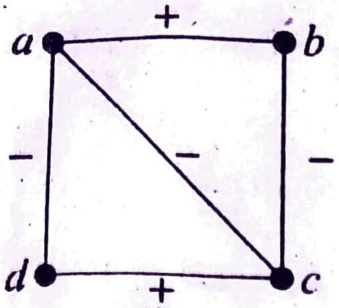
(iii) What is the average and maximum waiting time per ship?

(iv) What is the length of the longest queue? (6)

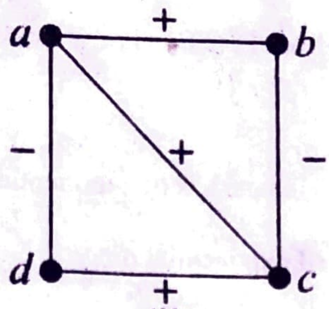
(8)

3(a) Define Isomorphism of graphs. Draw two non-isomorphic regular graphs with eight vertices and twelve edges. Justify your answer. (6)

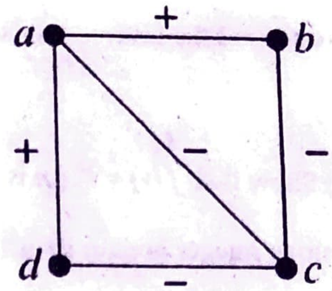
(b) Determine which of the following signed graphs are balanced. Also, find the corresponding bipartite graph in each of the cases. Justify your answer.



(i)



(ii)



(iii)

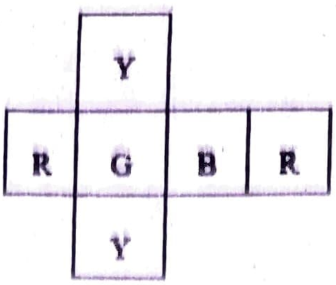
(6)

(c) Prove that, a connected graph is Eulerian if and only if each of its vertices has an even degree. (6)

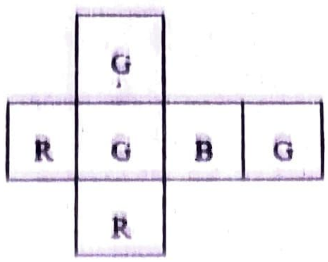
(d) (i) Define a Hamiltonian graph. For which values of r and s is the complete bipartite graph $K_{r,s}$ Hamiltonian? Justify. (3)

(ii) State Ore's Theorem. Give an example of a Hamiltonian graph that does not satisy the conditions of the Ore's Theorem. (3)

4. (a)Find the solution to the four-cubes problem of the following set of cubes:



Cube1



Cube 2

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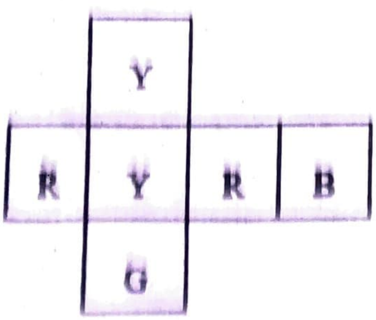
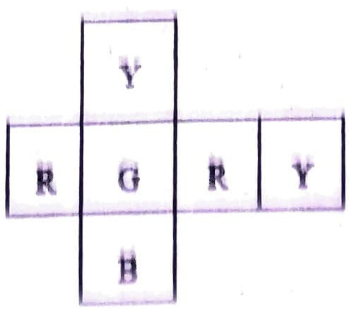


Cube 3



Cube 4

(b) Solve the following initial value problemin powers of $(t-1)$.

$$(t^2-2t-3)\frac{d^2y}{dt^2}=3(t-1)\frac{dy}{dt}+y=0;\ y(1)=4,\ y'(1)=-1$$ (7)

(c)(i)Find the inverse Laplace transform of the function $F(s)=\frac{s^2-2s}{s^4+5s^2+4}$. (4)

(ii) Show that $f(t)=t^n$ (n is a positive integer), is of exponential order, but $g(t)=e^{t^n}$ (n is a positive integer greater than 1) , is not of exponential order. (3)

(d) Solve the problem

Maximize $12x_1+7x_2$

subject to

$5x_1+4x_2\le 20$

$3x_1+5x_2\le 18$

$x_1,x_2\ge 0$

Determine the sensitivity of the optimal solution to a change in C_2 using objective function $12x_1+c_2x_2$. (7)

This question paper contains 3 printed pages.

Your Roll No. 7/12/17

Sl. No. of Ques. Paper: 5713

H

Unique Paper Code : 235503

Name of Paper : Analysis – IV

Name of Course : B.Sc. (Hons.) Mathematics

Semester : V

Duration : 3 hours

Maximum Marks : 75

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All questions are compulsory.
Attempt any two parts from each question.

1(*a*) Suppose (X, d) is a metric space and $f : X \to \mathbb{R}$. Prove that the function

$$e : X \times X \to \mathbb{R}^+ \cup \{0\}$$

defined by

$$e(a, b) = d(a, b) + |f(a) - f(b)|, \quad \forall\, a, b \in X$$

is a metric on X.

(*b*) Suppose S is a nonempty bounded above subset of $\mathbb{R}$ and $z \in \mathbb{R}$. Prove that

$$dist(z, S) \le |z - Sup(S)|$$

with equality if $z \ge Sup(S)$.

(*c*) Suppose (X, d) is a metric space, $x \in X$, A and B are nonempty subsets of X for which $A \subseteq B$. Prove that:

$$dist(x, B) \le dist(x, A) \le dist(x, B) + diam(B)$$

$\left(6\frac{1}{2} + 6\frac{1}{2}\right)$

2(*a*) Suppose (X, d) is a metric space and F is a finite subset of X. Show that $acc(F) = \phi$

(*b*) Suppose X is a metric space, A and B are subsets of X and $A \subseteq B$. Prove that:

(*i*) $\bar{A} \subseteq \bar{B}$

(ii) $A^o \subseteq B^o$

(c) Suppose (X, d) is a metric space, $w \in X$ and $A \subseteq X$. Prove that:

(i) $dist(w, A) \leq dist(w, \partial A)$ $\left(6\frac{1}{2} + 6\frac{1}{2}\right)$

(ii) $dist(w, \overline{A}) = dist(w, A)$

3(a) Suppose (X, d) is a metric space, $x \in X$ and S is a subset of X. Prove that $x \in \overline{S}$ if and only if every open ball of X centered at x has nonempty intersection with S.

(b) Suppose (X, d) is a metric space and A is a subset of X. Show that

(i) $A^o \cup \partial A = \overline{A}$

(ii) $\partial A = \overline{A} \cup \overline{A^c}$

(c) Suppose (X, d) is a metric space, $a \in X$ and $r \in \mathbb{R}^+$. Prove that:

(i) $\partial(b[a; r]) = \{x \in X \mid d(x, a) = r\}$ $\left(6\frac{1}{2} + 6\frac{1}{2}\right)$

(ii) $b[a; r]$ is closed in X.

4(a) Suppose (X, d) is a metric space and (x_n) is a Cauchy sequence in X. Prove that (x_n) converges in X if and only if (x_n) has a subsequence that converges in X.

(b) Suppose (X, d) is a metric space.

(i) Define a bounded set and a totally bounded set in (X, d).

(ii) Give an example of a subset A of X such that A is bounded in (X, d) but A is not totally bounded in (X, d).

(c)(i) Suppose (X, d) is a discrete metric space. Show that every function $f : (X, d) \to (Y, e)$ is a continuous function on X where (Y, e) is any metric space.

(ii) Show that every constant function from a metric space (X, d) to a metric space (Y, e) is continuous. $\left(6\frac{1}{2} + 6\frac{1}{2}\right)$

5(a) Suppose (X, d) is a complete metric space and (F_n) is a decreasing sequence of nonempty closed subsets of X such that $diam(F_n) \to 0$ as $n \to \infty$. Prove that

$$F = \bigcap_{n=1}^{\infty} F_n$$

contains exactly one point.

(*b*)(*i*) State Banach's Fixed Point Theorem.

(*ii*) Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ is a differentiable function and there exists $k \in (0,1)$ such that $|f'(x)| \leq k, \ \forall x \in \mathbb{R}$. Show that f has a unique fixed point.

(*c*) Prove that $\mathbb{R}$ (with usual metric) is connected.

$\left(6\frac{1}{2} + 6\frac{1}{2}\right)$

6(*a*) Suppose X is a metric space, S is a connected subset of X and $S \subseteq A \subseteq \overline{A}$. Prove that A is connected.

(*b*) Define a compact metric space. Give one example of each of the following:

(*i*) A metric space which is compact.

(*ii*) A metric space which is not compact.

(*c*) Suppose (X, d_1) and (Y, d_2) are metric spaces, (X, d_1) is compact and

$$f : (X, d_1) \longrightarrow (Y, d_2)$$

is continuous on X. Prove that f is uniformly continuous on X.

(5 + 5)

This question paper contains 3 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper: 5714 ③ H

Unique Paper Code : 235504

Name of Paper : Algebra – IV (MAHT-503)

Name of Course : B.Sc. (Hons.) Mathematics

Semester : V

Duration : 3 hours

Maximum Marks : 75

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Attempt any five parts from Q. No. 1.
Each part of Q. No. 1 carries 3 marks.
Attempt any two parts from each of the Questions Nos. 2 to 6. Each part carries 6 marks.

1. (a) Let $T: P_1(\mathbf{R}) \to \mathbf{R}^2$ be defined by $T(p(x)) = (p(0), p(2))$.
Find $[T^t]_{\gamma^*}^{\beta^*}$ where β and γ are the standard basis for $P_1(\mathbf{R})$ and $\mathbf{R}^2$ respectively.

(b) Prove that a linear operator T on a finite dimensional vector space V is invertible if and only if zero is not an eigen value of T.

(c) Let T be a linear operator on $P_3(\mathbf{R})$ defined by $T(f(x)) = f''(x)$.
Find an ordered basis for the T –cyclic subspace generated by $z = x^3$.

(d) Let β be a basis for a finite dimensional inner product space. Prove that if $\langle x, z\rangle = 0$ for all $z \in \beta$, then $x = 0$.

(e) Prove that $\mathbf{Q}(\sqrt{7}) = \mathbf{Q}(1 + \sqrt{7})$.

(f) Let a field K be an extension of field F. Suppose that E_1 and E_2 are fields contained in K and are extensions of F. If $[E_1: F]$ and $[E_2: F]$ are both prime, show that $E_1 = E_2$ or $E_1 \cap E_2 = F$.

(g) Let T be the linear operator on $\mathbf{R}^3$ defined by
$$T(x, y, z) = (4x + z, 2x + 3y + 2z, x + 4z)$$

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Find the eigen values and eigen vectors corresponding to any one of the eigen value of T.

Q2. (a) Let T be a linear operator on vector space V. Let $\lambda_1, \lambda_2, \dots, \lambda_k$ be distinct eigen values of T. If $v_1, v_2, \dots, v_k$ are eigen vectors of T such that λ_i corresponds to v_i $(i = 1, \dots, k)$, then prove that the set $\{v_1, v_2, \dots, v_k\}$ is linearly independent.

(b) Let T be a linear operator on vector space V. Let $\lambda_1, \lambda_2, \dots, \lambda_k$ be distinct eigen values of T. For each $i = 1, 2, \dots, k$, let S_i be a finite linearly independent subset of the eigen space E_{λ_i}. Show that $S = S_1 \cup S_2 \cup \dots \cup S_k$ is a linearly independent subset of V.

(c) Let $V = \mathbb{R}^3$, and, define $f_1, f_2, f_3 \in V^*$ by
$f_1((x, y, z)) = x - 2y$, $f_2((x, y, z)) = x + y + z$ and $f_3((x, y, z)) = y - 3z$.
Prove that $\{f_1, f_2, f_3\}$ is a basis for V^*, and find a basis for V for which it is the dual basis.

Q3. (a) Let T be a linear operator on a finite dimensional vector space V and W be T-cyclic subspace of V generated by non-zero vector $v \in V$. Let $\dim W = k$. Show that $\{v, T(v), T^2(v), \dots, T^{k-1}(v)\}$ is a basis for W.

(b) Let $V = P(\mathbb{R})$ with the inner product $\langle f(x), g(x) \rangle = \int_{-1}^{1} f(t)g(t)dt$ and $W = span\{1, x, x^2\}$ be the subspace of V. Find an orthogonal basis for W using Gram-Schmidt process.

(c) Let V be an inner product space, and let $S = \{v_1, v_2, \dots, v_n\}$ be an orthonormal subset of V. Prove that for any $x \in V$, $\|x\|^2 \geq \sum_{i=1}^{n} |\langle x, v_i \rangle|^2$.

Q4. (a) Let V be a finite dimensional inner product space over F and $g: V \to F$ be a linear transformation, prove that there exists a unique vector $y \in V$ such that $g(x) = \langle x, y \rangle$ $\forall\, x \in V$.

(b) Determine all $A \in M_{2\times 2}(\mathbb{R})$ for which $A^2 - 3A + 2I = 0$.

(c) Let $p(t)$ be the minimal polynomial of a linear operator T on a finite dimensional vecor space V. Prove the following:

(i) For any polynomial $g(t)$, if $g(T) = T_0$, then $p(t)$ divides $g(t)$. In particular, $p(t)$ divides the characteristic polynomial of T.

(ii) The minimal polynomial of T is unique.

Q5. (a) Prove or disprove that $\mathbb{Q}(\sqrt{3})$ and $\mathbb{Q}(\sqrt{-3})$ are field isomorphic.

(b) Let F be a field and let $p(x) \in F[x]$ be irreducible over F such that $\deg p(x) = n$. Let a be a zero of $p(x)$ in some extension E of F. Show the following:

(i) $F(a)$ is isomorphic to $\frac{F[x]}{\langle p(x) \rangle}$.

(ii) Every member of $F(a)$ can be uniquely expressed in the form $c_{n-1}a^{n-1} + c_{n-2}a^{n-2} + \dots + c_1 a + c_0$ where $c_0, c_1, \dots, c_{n-1} \in F$.

(c) Let K be a finite field extension of the field E and let E be a finite field extension of the field F. Show that K is a finite extension field field of the F and that $[K: F]=[K: E]\,[E: F]$.

Q6. (a) Let K be a finite extension of a finite field F. Show that there exists $a \in K$ such that $K = F(a)$.

(b) (i) Prove that $\cos\theta$ is constructible if and only if $\cos 2\theta$ is constructible.

(ii) Prove that $\sqrt{a}$ is constructible if $a > 0$ is constructible.

(c) If K is an algebraic extension of the field E and if E is an algebraic extension of the field F. Show that K is an algebraic extension of the field F.

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Your Roll No. ....

Sl. No. of Ques. Paper: 5715 (4) H

Unique Paper Code : 235505

Name of Paper : Lineer Programming and Theory of Games (MAHT-504)

Name of Course : B.Sc. (Hons.) Mathematics

Semester : V

Duration : 3 hours

Maximum Marks : 75

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Attempt any two parts from each question.
All questions carry equal marks.

1. (a) If the L.P.P. is

$$\text{Maximize } z = cx$$

subject to

$$Ax = b$$
$$x \geq 0$$

$(x_B, 0)$ is a basic feasible solution corresponding to basis B having an a_j with $z_j - c_j < 0$ and all corresponding $y_{ij} \leq 0$, then show that the L.P.P. has an unbounded solution.

(b) Consider the following linear programming problem:

Maximize $z = -3x_1 - 2x_2$

subject to

$$-x_1 + x_2 \leq 1$$
$$5x_1 + 3x_2 \leq 15$$
$$x_1 \geq 0$$
$$x_2 \geq \frac{3}{2}.$$

Solve the problem graphically.

(c) Consider the following system

$$x_1 + x_2 + x_3 \leq 2$$
$$-x_1 + 2x_2 + 2x_3 \leq 3$$
$$x_1,\ x_2,\ x_3 \geq 0$$

The point $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ is feasible. Verify whether it is basic. If not, then reduce it to a basic feasible solution.

2. (a) Solve the following problem by simplex method:

Maximize $z = x_1 - 2x_2 + x_3$

subject to

$$x_1 + 2x_2 + x_3 \le 12$$
$$2x_1 + x_2 - x_3 \le 6$$
$$-x_1 + 3x_2 \le 9$$
$$x_1,\ x_2,\ x_3 \ge 0.$$

(b) Use Big *M* method to solve the following linear programming problem:

Maximize $$z = 6x_1 + 4x_2$$

subject to the constraints

$$2x_1 + 3x_2 \le 30$$
$$3x_1 + 2x_2 \le 24$$
$$x_1 + x_2 \ge 3$$
$$x_1,\ x_2 \ge 0.$$

(c) Show that there does not exist any feasible solution to the linear programming problem:

Maximize $$z = 2x_1 + 3x_2 + 5x_3$$

subject to the constraints

$$3x_1 + 10x_2 + 5x_3 \le 15$$
$$33x_1 - 10x_2 + 9x_3 \le 33$$
$$x_1 + 2x_2 + x_3 \ge 4$$
$$x_1,\ x_2,\ x_3 \ge 0.$$

3. (a) If $(x_B, 0)$ is a primal optimal solution, then show that there exists a dual optimal solution w, such that $c_B x_B = b^T w$.

(b) Maximize $$z = x_1 - x_2 + 3x_3 + 2x_4$$

subject to the constraints

$$x_1 + x_2 \ge -1$$
$$x_1 - 3x_2 - x_3 \le 7$$
$$x_1 + x_3 - 3x_4 = -2$$
$$x_1,\ x_4 \ge 0$$
$$x_2,\ x_3 \text{ unrestricted}$$

(c) Use duality to solve the following L.P.P.:

Maximize $$z = 2x_1 + x_2$$

subject to the constraints

$$x_1 + 2x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$x_1 - 2x_2 \le 1$$
$$x_1,\ x_2 \ge 0$$

4. (a) Solve the following assignment problem:

	I	II	III	IV	V
A	3	5	10	15	8
B	4	7	15	18	8
C	8	12	20	20	12
D	5	5	8	10	6
E	10	10	15	25	10

(b) Solve the following transportation problem:

	D_1	D_2	D_3	Availability
P_1	300	360	425	600
P_2	390	340	310	300
P_3	255	295	275	1000
Requirement	400	500	800	

(c) Solve the following game graphically:

Player B

Player A $\begin{bmatrix} 1 & 2 \\ 5 & 4 \\ -7 & 9 \\ -4 & -3 \\ 2 & 1 \end{bmatrix}$

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5. (a) Consider the game with the following payoff matrix:

Player B

$$\text{Player } A \quad \begin{matrix} & B_1 & B_2 \\ A_1 & 2 & 6 \\ A_2 & -2 & \lambda \end{matrix}$$

(i) Show that the game is strictly determinable, whatever λ may be.

(ii) Determine the value of the game.

(b) Solve the following game:

	I	II	III	IV
I	3	2	4	0
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

(c) Convert the following game, involving two-person, zero-sum game, into a linear programming problem and hence solve it.

Player B

Player A $\begin{bmatrix} 5 & 7 & 2 \\ 10 & 4 & 9 \\ 6 & 2 & 0 \end{bmatrix}$

04/12/17

Sl. No. of Q.P.: 6083

Unique Paper Code: 2351501

Name of the Paper: Algebra - IV (Group Theory- II)

Name of the Course: B.Sc. (Hons.) Mathematics- III (~~Erstwhile FYUP~~)

F-9

Semester: V

(5)

Duration: 3 Hours

Maximum Marks: 75

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Instructions for Candidates

1. Attempt any two parts from each question.
2. All questions are compulsory.

Q1. (a) Find Aut(Z_{10}). Also make its Cayley table.

(b) Define inner automorphism induced by an element g of a group G. If g and h induce the same inner automorphism of a group G, prove that $h^{-1}g \in Z(G)$, the center of G.

(c) For any group G, Prove that $G/Z(G)$ is isomorphic to Inn(G), where Inn(G) is the group of all inner automorphisms of G.

(6.5, 6.5, 6.5)

Q2. (a) Define characteristic subgroup of a group. Prove that the center of a group G is a characteristic subgroup of G.

(b) Let G' be the commutator subgroup of a group G. Prove that G' is normal in G. Also prove that G/G' is Abelian.

(c) Let G and H be finite cyclic groups. Prove that $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime.

(6, 6, 6)

Q3. (a) Prove that if a group G is the internal direct product of finite number of its subgroups $H_1, H_2, \ldots, H_n$, then G is isomorphic to the external direct product of $H_1, H_2, \ldots, H_n$.

(b) Express $U(105)$ as an external direct product of cyclic additive groups of the form Z_n.

(c) Find all Abelian groups (upto isomorphism) of order 72.

(6.5, 6.5, 6.5)

Q4. (a) Define group action of a group G on a set A. Prove that the additive group Z of integers acts on itself by $z.a = z + a$ for all $z, a \in Z$. Also find kernel of this action.

(b) Let G be a group. Define centralizer $C_G(A)$ and normalizer $N_G(A)$, of a non – empty subset A of G. Also prove that $C_G(Z(G)) = G$ and deduce that $N_G(Z(G)) = G$, where $Z(G)$ is the center of G.

(c) Let $G = \{1, a, b, c\}$ be the Klein 4 – group. Label $1, a, b, c$ with the integers 1, 2, 3, 4 respectively. Compute the image of each element of G under the left regular representation of G into S_4.

Space (6, 6, 6)

Q5. (a) If p is a prime and P is a group of prime power order p^{α} for some $\alpha \geq 1$ then prove that P has a non – trivial center : $Z(P) \neq 1$.

(b) Determine whether σ_1 and σ_2 are conjugate. If they are, give permutation τ such that $\tau\sigma_1\tau^{-1} = \sigma_2$.

(i) $\sigma_1 = (12)(345)$, $\sigma_2 = (123)(45)$.

(ii) $\sigma_1 = (13)(246)$, $\sigma_2 = (35)(24)(56)$.

(c) Make the list of partitions of $n = 5$ and give representative for the corresponding conjugacy classes of S_5.

(6.5, 6.5, 6.5)

Q6. (a) State index theorem. Use this to prove that there is no simple group of order 216.

(b) Exhibit all Sylow 3 – subgroup of A_4.

(c) Prove that a group of order 56 has a normal Sylow p – subgroup, for some prime p dividing its order.

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(6, 6, 6)

Sr. No. of Question Paper : 6084

Roll No...............

Unique Paper Code : 2351502

Name of the Course : B.Sc. (H) Mathematics-III(~~Erstwhile FYUP~~)

Name of the Paper : Analysis-V(Complex Analysis)

Semester : V

Duration : 3 Hours

Maximum Marks: 75

Instructions for Candidates

1. *Write your Roll No. on the top immediately on receipt of this question paper.*
2. *Attempt any five parts from Question No. 1 and any two parts from each of Question Nos. 2 to 6.*
3. *All questions are compulsory.*

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1. Attempt **any five** parts. Each part carries 3 marks.

(a) Write the function $f(z) = z^3 + z + 1$ in the form $f(z) = u(x, y) + iv(x, y)$.

(b) Find $\lim_{z \to 1-i} [x + i(2x + y)]$, where $z = x + iy$.

(c) Prove that the function $f(z) = z^2$ is differentiable everywhere.

(d) Suppose that a function $f(z) = u(x, y) + i\,v(x, y)$ and its conjugate function are analytic in a given domain D. Prove that the function $f(z)$ must be constant throughout the domain D.

(e) Show that $\exp(2 \pm 3\pi i) = -e^2$.

(f) Find the residue at $z = 0$ of the function $f(z) = \frac{z - \sin z}{z}$.

(g) Prove that $z = 0$ is a removable singular point of the function $f(z) = \frac{1 - \cos z}{z^2}$.

2. (a) (i) Find the value of $\lim_{n \to \infty} \left(\frac{1}{n^3} + 1\right)$.

(ii) Let $\theta_n \, (n = 1, 2, 3, \ldots)$ denotes the principal arguments of the numbers

$$z_n = 2 + i\frac{(-1)^n}{n^2} \quad (n = 1, 2, \ldots)$$

Then prove that $\lim_{n \to \infty} \theta_n = 0$. (3,3)

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(b) If a function $f(z)$ and its conjugate $\overline{f(z)}$ both are analytic in a domain D, then show that $f(z)$ is constant throughout D. (6)

(c) (i) Show that if e^z is real, then $\operatorname{Im} z = n\pi\ (n = 0, \pm1, \pm2, \pm3, \ldots)$.

(ii) Find all singular points of the function $f(z) = \frac{1}{z(e^z-1)}$. (3,3)

3. (a) State and prove Cauchy-Riemann equations for the function $f(z) = u(x,y) + iv(x,y)$ at the point $z_0 = (x_0, y_0)$ where f' exists. (6)

(b) (i) Show that $Log(i^3) \neq 3Log(i)$ where $Log(z)$ is the principal branch of $\log z$.

(ii) Find all values of z such that $exp(2z-1)=1$. (6)

(c) Find all roots of the equation $\sin z = \cosh 4$. (6)

4. (a) Without evaluating the integral show that $\left|\int_C \frac{dz}{z^2-1}\right| \leq \frac{\pi}{3}$ where C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$. (6)

(b) Evaluate the following integrals:

(i) $\int_C f(z)dz$ where $f(z) = \frac{z+2}{z}$ and C is the semicircle $z = 2e^{i\theta}\ (0 \leq \theta \leq \pi)$.

(ii) $\int_C \frac{dz}{z^2+4}$ where C is the circle $|z - i| = 2$ in the positive sense. (6)

(c) State and prove Cauchy Integral Formula. (6)

5. (a) Using the Maclaurin's series expansion for the entire function e^z and the definition of $Sin z$, find the Maclaurin's series expansion of the function $Sin z$. (6)

(b) Write down the Laurentz's series expansion of the function $f(z) = \frac{1}{z(z-1)}$ in the domain $|z| > 1$. (6)

(c) If z_1 is a point inside the circle of convergence $|z - z_0| = R$ of a power series $\sum_{n=0}^{\infty} a_n(z-z_0)^n$, then show that the series must be uniformly convergent in the closed disk $|z - z_0| \leq R_1$, where $R_1 = |z_1 - z_0|$. (6)

6. (a) Evaluate the integral $\int_C z^2 \sin\left(\frac{1}{z}\right) dz$, where C is the positively oriented unit circle $|z|=1$.

(6)

(b) Find out the sum of the residues of the function $f(z)=\frac{z+1}{z^2+9}$ calculated on its isolated singular points.

(6)

(c) Evaluate $\int_0^{2\pi} \frac{\cos^2 3\theta}{5-4\cos 2\theta} d\theta$.

(6)

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Sl-No of Q.P : 6085

19/12/17

Unique Paper Code	:	2351503
Name of the Paper	:	Calculus II (Multivariate Calculus)
Name of the Course	:	B.Sc.(H) Mathematics
Semester	:	V
Duration	:	3 hours (7)
Maximum Marks	:	75

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Instructions for Candidates

(i) Attempt any five questions from each section.
(ii) Each question carries 5 marks.
(iii) Use of scientific calculators is allowed.

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Section-I

1. Determine the standard form of equation for the tangent plane to the surface $z = 10 - x^2 - y^2$ at $P_0(2,2,2)$.

2. a) Find the slope of the line that is parallel to the xz-plane and tangent to the surface $z = x\sqrt{x+y}$ at the point P(1,3,2).

   b) Let $f(x,y) = \begin{cases} xy(x^2 - y^2), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

   Is $f_{xy}(0,0) = f_{yx}(0,0)$?

3. The radius and height of a right circular cone are measured with errors of at most 3% and 2%, respectively. Use increments to approximate the maximum possible percentage error in computing the volume of the cone using the formula $V = \frac{1}{3}\pi R^2 H$.

4. Find the directional derivative of $f(x,y) = 3 - 2x^2 + y^3$ at the point $P(1,2)$ in the direction of the unit vector $u = \frac{1}{2}i - \frac{\sqrt{3}}{2}j$.

5. Given that the largest and smallest values of $f(x,y) = 1 - x^2 - y^2$ subject to the constraint $x + y = 1$ with $x \geq 0$, $y \geq 0$ exist, use the method of Lagrange multipliers to find these extrema.

6. Find all the points on the surface $y^2 = 4 + xz$ that are closest to the origin.

Section II

7. Compute the area of the region D bounded above by the line y = x and below by the circle $x^2 + y^2 - 2y = 0$.

8. Find the volume of the tetrahedron T bounded by the plane 2x + y + 3z = 6 and the coordinate planes x = 0, y = 0 and z = 0.

9. Find the area of the region E bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

10. Find the Volume of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$.

11. A wire has the shape of the curve $x = \sqrt{2}\sin(t)$, $y = \cos(t)$, $z = \cos(t)$ for $0 \le t \le \pi$. If the wire has density $\delta(x, y, z) = xyz$ at each point (x, y, z), What is its mass? what

12. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\, dydx$ by converting to polar coordinates.

Section III

13. State Green's Theorem and verify it for the line integral $\oint(-y\,dx + x\,dy)$, over the curve $C = C_1 \cup C_2$, where C_1 is the line segment from (-1, 0) to (1, 0) and C_2 is a semi-circular arc from (1, 0) back to (-1, 0).

14. Show that the vector field $\vec{F} = (20x^3z + 2y^2, 4xy, 5x^4 + 3z^2)$ is conservative in $\mathbb{R}^3$ and find a scalar potential for $\vec{F}$.

15. Evaluate the surface integral $\iint_S g\,dS$, where $g(x,y) = xy + 2x^2 - 3xy$ and S is that portion of the plane $2x - 3y + z = 6$ that lies over the unit square $R: 2 \le x \le 3, 2 \le y \le 3$.

16. Evaluate $\iint_S (x + y + z)\,dS$, where S is the surface defined parametrically by $R(u, v) = (2u + v)i + (u - 2v)j + (u + 3v)k$ for $0 \le u \le 1, 0 \le v \le 2$.

17. Evaluate $\oint_C (\frac{1}{2}y^2dx + zdy + xdz)$, where C is the curve of intersection of the plane $x + z = 1$ and the ellipsoid $x^2 + 2y^2 + z^2 = 1$, oriented counter clockwise as viewed from above.

18. a) Let F be a constant vector field. What is
    i) div F
    ii) curl F

    b) Show that $f(x,y) = e^x\cos(y)$ is harmonic.

This question paper contains 8 printed pages]

30/11/17

Roll No. [ ]

S. No. of Question Paper : **6626** (8)

Unique Paper Code : **32351501** **HC**

Name of the Paper : **C11-Metric Spaces**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

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Attempt any *two* parts from each question.

All questions are compulsory.

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1. (*a*) Let $d_p (p \geq 1)$ on the set $\mathbf{R}^n$, be given by

$$d_p(x, y) = \left(\sum_{j=1}^{n} |x_j - y_j|^p \right)^{1/p},$$

for all $x = (x_1, x_2, \ldots\ldots, x_n), y = (y_1, y_2, \ldots\ldots, y_n)$ in $\mathbf{R}^n$.

Show that $(\mathbf{R}^n, d_p)$ is a metric space. Does d_p define a metric on $\mathbf{R}^n$, when $0 < p < 1$? 4+2=6 (*b*) Let S be any non-empty set and B(S) denote the set of all real- or complex-valued functions on S, each of which is bounded. Define the uniform metric d on B(S). Show that (B(S), d) is a complete metric space. 6

(*c*) (*i*) Let X = N, the set of natural numbers. Define

$$d(m, n) = \left|\frac{1}{m} - \frac{1}{n}\right|, \; m, n \in X.$$

Show that (X, d) is an incomplete metric space.

(*ii*) Prove that metric spaces, **R** with the usual metric and $(0, \infty)$ with the usual metric induced from **R** are homeomorphic. 4+2=6

2. (*a*) (*i*) Let (X, d) be a metric space. Prove that the closed ball $\bar{S}(x, r)$, where $x \in X$ and $r > 0$, is a closed subset of X.

(*ii*) Is the set $A = \{(x, y) : x + y = 1\}$ open in the metric space (R^2, d_2) ? Justify your answer. 4+2=6

(*b*) Let (X, d) be a metric space and Y a subspace of X. Let Z be a subset of Y. Prove that Z is closed in Y if and only if there exists a closed set F of X such that $X = F \cap Y$. 6

(*c*) (*i*) Let (X, d) be a metric space and F_1 and F_2 be subsets of X. Prove that :

$$cl(F_1 \cup F_2) = cl(F_1) \cup cl(F_2).$$

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(*ii*) Define a separable metric space. Is the discrete metric space (X, d) separable ? Justify your answer. 3+3=6

3. (*a*) Let (X, d) be a metric space and for every nested sequence $\{F_n\}$, $n \geq 1$ of non-empty closed subsets of X such that $d(F_n) \to 0$ as $n \to \infty$, the intersection $\bigcap_{n=1}^{\infty} F_n$

contains one and only one point. Prove that (X, d) is a complete metric space. Further, show that the condition : $d(F_n) \to 0$ as $n \to \infty$ in the above statement can't be dropped. 4+2=6

(*b*) Let (X, d) be a metric space and F $\subseteq$ X. Prove that a point x_0 is a limit point of F if and only if it is possible to select from the set F a sequence, $\{x_n\}$, $n \geq 1$, of distinct points such that $\lim_n d(x_n, x_0) = 0$. 6

(*c*) (*i*) Let F be subset of the metric space (X, d). Prove that the set of limit points of F is a closed subset of (X, d).

(*ii*) Let F be a non-empty bounded closed subset of **R**, with usual metric and a = sup F. Show that $a \in$ F. 3+3=6

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4. (*a*) (*i*) Let (X, d) be any metric space and $f : (X, d) \to (R^n, d_2)$, be defined by :

$f(x) = (f_1(x), f_2(x), \ldots\ldots, f_n(x))$, for $x \in$ X.

Show that if f is continuous, so is each $f_k : X \to \mathbf{R}$, $k = 1, 2, \ldots\ldots, n$.

(*ii*) Let (X, d_X) and (Y, d_Y) be metric spaces and let $f : X \to Y$ be continuous on X. Show that $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$, for all subsets B of Y. 2½+4=6½

(*b*) Let (X, d_X) and (Y, d_Y) be metric spaces and let $f : X \to Y$ be uniformly continuous. Show that if $\{x_n\}$, $n \geq 1$, is a Cauchy sequence in X, then so is $\{f(x_n)\}$, $n \geq 1$, in Y. Is this result true, if $f : X \to Y$ is continuous on X ? 4+2½=6½

(c) Let X be the set of all continuous functions defined on [0, 1]. For $f, g \in X$, define the metrics 'd' and 'e' on X by :

$d(f, g) = \sup\{|f(x) - g(x)| : x \in [0, 1]\}$, and

$e(f, g) = \int_0^1 |f(x) - g(x)|\, dx.$

Show that these metrics are not equivalent. 6½

5\. (a) (i) Let (X, d) be the complete metric space and $T : X \to X$ be a contraction mapping and let $x_0 \in X$ and $\{x_n\}$, $n \geq 1$, be the sequence defined iteratively by $x_{n+1} = T x_n$ for $n = 0, 1, 2, \ldots$.

Show that the sequence $\{x_n\}$, $n \geq 1$, is convergent in X.

(ii) Let $T : X \to X$, where (X, d) is a complete metric space, satisfy the inequality :

$d(Tx, Ty) < d(x, y)$ for all $x, y \in X$.

Show that T need not have a fixed point.

4+2½=6½

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(b) Let (**R**, d) be the space of real numbers with the usual metric. Show that a subset, I, of **R** is connected if and only if I is an interval. 6½

(c) (i) Show that the metric space (X, d) is disconnected if and only if there exists a proper subset of X that is both open and closed in X.

(ii) Let A be a subset of $\mathbf{R}^2$ defined by

$A = \{(x, y) : x^2 - y^2 \geq 4\}$.

Show that A is disconnected. 4+2½=6½

6. (*a*) Let (X, d_X) be a metric space and every continuous function $f:(X, d_X) \rightarrow (\mathbf{R}, d_u)$, where d_u is the usual metric of $\mathbf{R}$, has the intermediate value property. Prove that (X, d_X) is a connected space. 6½

(*b*) Define the finite intersection property. Prove that a metric space (X, d) is compact if, and only if every collection of closed sets in (X, d) with the finite intersection property has non-empty intersection. 6½

(*c*) Let f be a continuous function from a *compact* metric space (X, d_X) into a metric space (Y, d_Y). Prove that the range $f(X)$ is also compact. 6½

This question paper contains 4 printed pages. 5/12/17

Your Roll No. ....

Sl. No. of Ques. Paper: 6627 (9) **HC**

Unique Paper Code : **32351502**

Name of Paper : **Group Theory – II**

Name of Course : **B.Sc. (Hons.) Mathematics**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

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All questions are compulsory. Question No. 1 has been divided into 10 parts and each part is of 1.5 marks. Each question from Q. Nos. 2 to 6 has 3 parts and each part is of 6 marks. Attempt any two parts from each question.

(1) State true (T) or false (F). Justify your answer in brief.

(a) $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ is isomorphic to $\mathbb{Z}_4$ here $\mathbb{Z}_n$ is used for group $\{0, 1, 2, \ldots, n-1\}$under the addition modulo n..

(b) The dihedral group D_8 of order 8 is not isomorphic to the quaternion group Q_8 of order 8.

(c) The external direct product $G \oplus H$ is cyclic if and only if groups G and H are cyclic.

(d) $U(165)$ can be written as an external direct product of cyclic additive groups of the form $\mathbb{Z}_n$ where $U(n)$ denotes the group of units under multiplication modulo n.

(e) Translations $z \mapsto z + a$ are the only automorphisms of the additive group of integers $\mathbb{Z}$.

(f) A subgroup N of a group G is called a characteristic subgroup if $\phi(N) = N$ for all isomorphism of G onto itself.

(g) The number of isomorphism types (classes) of a group of order 9 is 3.

(h) If G is a finite group of order n, then G is isomorphic to a subgroup of $D_n n$.

(i) If a group G acts trivially on a set A containing more than 1 elements then there is an element a in A whose stabilizer is proper subgroup of the group.

(j) $U(8)$ is isomorphic to $U(10)$.

(2) (a) Find all subgroups of order 3 in $\mathbb{Z}_9 \oplus \mathbb{Z}_3$.

(b) Prove that for any group G, $G/Z(G)$ is isomorphic to the group of inner automorphism $Inn(G)$ where $Z(G)$ is centre of the group G.

(c) Classify groups of order 6.

(3) (a) (i) Suppose that G is a group of order 4 with identity e and $x^2 = e$ for all x in G. Prove that G is isomorphic to $\mathbb{Z}_2 \oplus \mathbb{Z}_2$.

(ii) Find two groups G_1 and G_2 such that G_1 is isomorphic to G_2 but $Aut(G_1)$ is not isomorphic to $Aut(G_2)$ where $Aut(G_i)$ is the group of automorphisms of G_i.

(b) (i) Suppose that N is a normal subgroup of a finite group G. If G/N has an element of order n, show that G has an element of order n. Also show, by an example, that the assumption that G is finite is necessary.

(ii) If G is a non abelian group then show that $Aut(G)$ is not cyclic.

(c) Define the characteristic subgroup of a group G. Prove that every subgroup of a cyclic group is characteristic.

(4) (a) If p is a prime and G is a group of prime power order p^α for some positive integer $\alpha \geq 1$, then show that G has a non trivial centre.

(b) Find all conjugacy classes of the dihedral group D_8 of order 8 and the quaternion group Q_8 of order 8 and hence verify the class equation.

(c) Prove that if H has a finite index n in G then there is a normal subgroup K of G where K is subgroup of H and the index of K in G ($|G : K|$) is less than or equal to $n!$.

(5) (a) Prove that if p is a prime and G is a group of order p^α for some positive integer α then every subgroup of index p is normal in G. Deduce that every group of order p^2 has a normal subgroup of order p.

(b) Prove that a group of order 56 has a normal Sylow p-subgroup for some prime p dividing its order.

(c) Prove that two elements of the symmetric group on n letters S_n are conjugate in S_n if and only if they have same cycle type.Also show that the number of conjugacy classes equals the number of partitions of n.

(6) (a) Define a simple group. Prove that if G is an abelian simple group then G is isomorphic to the cyclic group $\mathbb{Z}_p$ for some prime p.

(b) (i) Prove that group of order 280 is not simple.

(ii) Show that the alternating group A_5 of degree 5 can not contain a subgroup of order 30 or 20 or 15.

(c) (i) If the centre of G is of index n, then prove that every conjugacy class has at most n elements.

(ii) Prove that the centre $Z(S_n)$ of symmetric group S_n contains only the identity of S_n for all n greater than or equal to three.

This question paper contains 6 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : **8508** **HC**

Unique Paper Code : **32357501**

Name of Paper : **Numerical Methods**

Name of Course : **Mathematics : DSE for Honours**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

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Use of non-programmable scientific calculator is allowed.

*Attempt **all** questions, selecting **two** parts from each question.*

1. (a) Give the geometrical construction of the Newton's method to approximate a zero of a function. Write an algorithm to find a root of $f(x) = 0$ by Newton's method.

   (b) Define order of convergence of an iterative scheme $\{x_n\}$. Determine the order of convergence for the recursive scheme:

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right).$$

(c) Define the rate of convergence of an iterative scheme $\{x_n\}$. Use the bisection method to determine the root of the equation $x^5 + 2x - 1 = 0$ on $(0, 1)$. Further, compute the theoretical error bound at the end of fifth iteration and the next enclosing (bracketing) interval. 13

2. (a) Differentiate between the method of false position and the secant method. Apply the method of false position to $\cos x - x = 0$ to determine an approximation to the root lying in the interval $(0, 1)$ until the absolute error is less than 10^{-3} $(p = 0{\cdot}739085)$.

(b) Let g be a continuous function on the closed interval $[a, b]$ with $g : [a, b] \to [a, b]$. Furthermore, suppose that g is differentiable on the open interval (a, b) and there exists a positive constant $k < 1$ such that $|g'(x)| \leq k < 1$ for all x belongs to (a, b). Then:

(i) The sequence $\{p_n\}$ generated by $p_n = g(p_n - 1)$ converges to the fixed point p for any p_0 belonging to $[a, b]$;

(ii) $|p_n - p_{n-1}| \leq k^n \max(p_0 - a, b - p_0)$.

(c) Find the approximated root of $f(x) = x^3 + 2x^2 - 3x - 1$ by secant method, taking $p_0 = 2$ and $p_1 = 1$ until $|p_n - p_{n-1}| < 5 \times 10^{-3}$. 13

3. (a) Using scaled partial pivoting during the factor step, find matrices L, U and P such that $LU = PA$ where

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}.$$

Hence, solve the system $Ax = b$, given

$$b = \begin{bmatrix} 14 \\ 36 \\ 7 \end{bmatrix}.$$

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(b) Use Jacobi method to solve the following system of linear equations. Use the initial approximation $x^{(0)} = 0$ and perform three iterations.

$$\begin{aligned} 4x_1 + 2x_2 - x_3 &= 1 \\ 2x_1 + 4x_2 + x_3 &= -1, \\ -x_1 + x_2 + 4x_3 &= 1. \end{aligned}$$

(c) (i) Consider the matrix:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{bmatrix}.$$

Find a lower triangular matrix L and an upper triangular matrix U with ones along its diagonal such that $A = LU$.

(ii) Determine the spectral radius of the matrix:

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}.$$ 13

4. (a) (i) If $x_0, x_1, x_2, \ldots\ldots x_{n+1}$ are $n+1$ distinct points and f is defined at $x_0, x_1, x_2, \ldots\ldots x_n$, then prove that interpolating polynomial P, of degree at most n, is unique.

(ii) Define the shift operator E and central difference operator δ. Prove that:

$$E = 1 + \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}.$$

(b) For the function $f(x) = e^x$, construct the Lagrange form of interpolating polynomial of f passing through the points $(-1, e^{-1})$, $(0, 1)$ and $(1, e)$. Estimate $\sqrt{e}$ using the polynomial. What is the error in the approximation? Verify that theoretical error bound is satisfied.

(c) (i) Write the following data in the usual divided difference tabular form and determine the missing values:

$$x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 3,$$

$$f[x_0] = 2,\ f[x_1] = 6,\ f[x_2] = 6,$$

$$f[x_0, x_1] = 4,\ f[x_2, x_3] = 0,\ f[x_1, x_2, x_3] = 0.$$

(ii) Prove that:

$$\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0.$$ 12

5. (a) Use the formula:

$$f'(x) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

to approximate the derivative of the function $f(x) = 1 + x + x^3$ at $x_0 = 1$, taking $h = 1, 0.1, 0.01$, and 0.001. What is the order of approximation?

(b) Verify:

$$f'(x) = \frac{-3f(x_0) + 4f(\pm h) - f(x_0 + 2h)}{2h},$$

the difference approximation for the first derivative provides the exact value of the derivative regardless of h, for the functions $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$, but not for the function $f(x) = x^3$.

(c) Derive second-order forward difference approximation to the first order derivative of a function. 12

6. (a) Approximate the value of the integral $\int_1^2 \frac{1}{2} dx$ using Simpson rule. Further verify the theoretical error bound.

(b) Apply Euler's method to approximate the solution of the given initial value problem $x' + \frac{4}{t} = t^4, (1 \le t \le 3), x(1) = 1, N = 5$. Further it is given that the exact solution is $x(t) = \frac{1}{9}(t^5 + 8t^{-4})$. Compute the absolute error at each step.

(c) Consider the initial value problem

$$x' = 1 + \frac{x}{t}, (1 \le t \le 3), x(1) = 1$$

whose exact solution is given by $x(t) = t(1 + \ln t)$. Using the step-size of 0.5, obtain the solution of the IVP and compare the absolute error with theoretical error bound, assuming the Lipschitz constant L equals 1. 12

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Your Roll No. 1911217

Sl. No. of Ques. Paper: 8512

Unique Paper Code : 32357505 (11) HC

Name of Paper : Discrete Mathematics

Name of Course : Mathematics : DSE for Hons.

Semester : V

Duration : 3 hours

Maximum Marks : 75

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Attempt any two parts from each question.

Section I

1 (a) Give an example of each of the following:

(i) An ordered set with exactly one maximal element but no greatest element.

(ii) An ordered set having more than one minimal element but no maximal element.

(6)

(b) (i) Let N_0 be the set of whole numbers equipped with the partial order $\le$ defined by $m \le n$ if and only if m divides n.

(ii) Draw a Hasse diagram for the subset $Q = \{1,2,4,5,6,12,20,30,60\}$ of $(N_0, \le)$. Find elements $a,b,c,d \in Q$ such that $a \vee b$ and $c \wedge d$ do not exist in Q. (6)

(c) Let L and M be two ordered sets and let f be an order-isomorphism from L onto M. Prove that if L is a lattice, then M is also a lattice and f is a lattice isomorphism.

(6)

2 (a) Let L_1 and L_2 be two bounded lattices. Define a relation $\le$ on their Cartesian product $L = L_1 \times L_2$ by $(a_1, a_2) \le (b_1, b_2)$ if and only if $a_1 \le b_1$ in L_1 and $a_2 \le b_2$ in L_2. Prove that $(L, \le)$ is a bounded lattice. (6.5)

(b) (i) Define a sublattice of a lattice. Prove that every interval in a lattice L is a sublattice of L.

(ii) Prove that a lattice L is a chain if and only if every non-empty subset of L is a sublattice of L.

(3.5, 3)

(c) Prove that in a lattice L, the following statements are true

(i) $x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)\ \forall x, y, z \in L$

(ii) $y \leq z \Rightarrow x \vee y \leq x \vee z\ \forall x, y, z \in L$

(4, 2.5)

Section II

3 (a) Define a modular lattice. Prove that any homomorphic image of a modular lattice is modular.

(6)

(b) Use the Quine-McCluskey method to find a minimal form of

$$xyz + xy'z + xy'z' + x'yz + x'yz' + x'y'z'.$$

(6)

(c) (i) Express $x + y'$ in disjunctive normal form in three variables x, y, z.

(ii) Change the following expression from disjunctive normal form to conjunctive normal form:

$$xy + x'y + x'y'.$$

(3, 3)

4 (a) (i) Prove that $(x \wedge y)' = x' \vee y'$ *and* $(x \vee y)' = x' \wedge y'$ for all x, y in a Boolean algebra B.

(ii) Show that the lattice ({1,2,3,6,9,18}, gcd, lcm) does not form a Boolean algebra for the set of positive divisors of 18.

(3.5, 3)

(b) Using the Karnaugh Diagrams, find at least one minimum form for

$$p = (x_1 + x_2)(x_1 + x_3) + x_1 x_2 x_3.$$

(6.5)

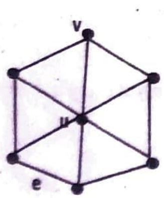
(c) Draw a switching circuit for a voting machine for a committee of three members. Here the motion is passed if and only if two or more switches are ON.

(6.5)

Section III

5 (a)(i) Prove that the number of odd vertices in a pseudograph is even.

(ii) Draw pictures of the subgraphs G \{e}, G \{v} and G \{u} of the following graph G.

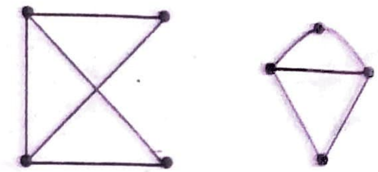


G

(3,3)

(b) (i) Draw K_4 and $K_{3,5}$.

(ii) For the below pair of graphs, either label the graphs so as to exhibit an isomorphism or explain why graphs are not isomorphic.



(3,3)

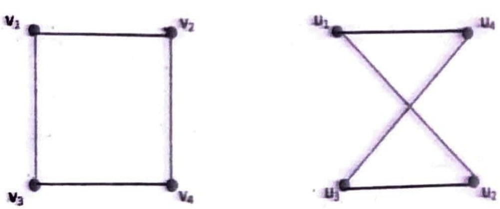
(c) (i) A complete graph with more than two vertices is not bipartite. Justify this statement

(ii) Draw a graph whose degree sequence is 3,3,2,2,2.

(iii) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 6. Justify your answer.

(2,2,2)

6 (a) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$.

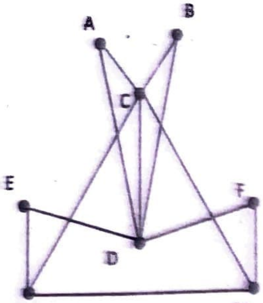


G_1 G_2

(6.5)

(b) (i) Consider the Graph G given below. Is it Hamiltonian? If no, explain your answers, if yes, find a Hamiltonian cycle.

(ii) Is it Eulerian? Explain.



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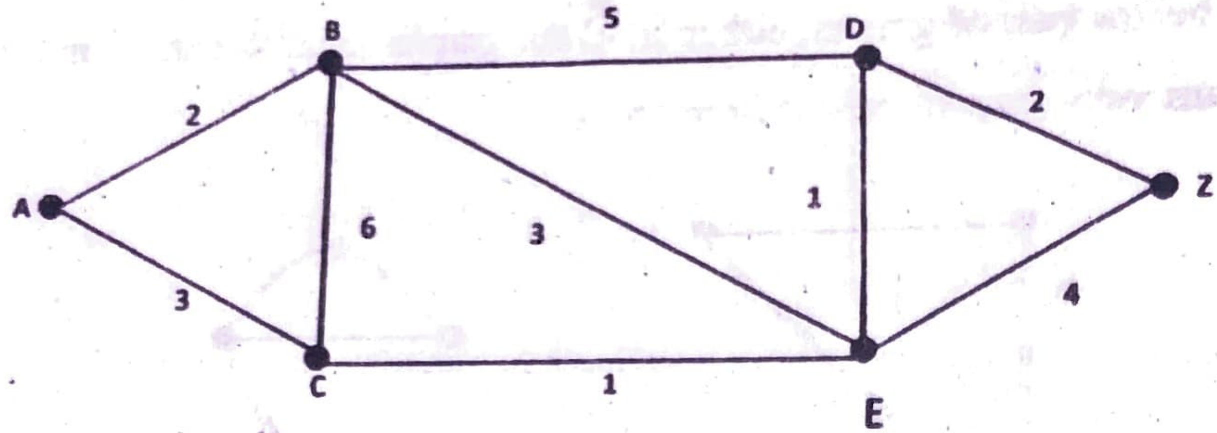
(6.5)

(c) Apply the improved version of Dijkstra's Algorithm to find a shortest path from A to Z. Write steps.



(6.5)