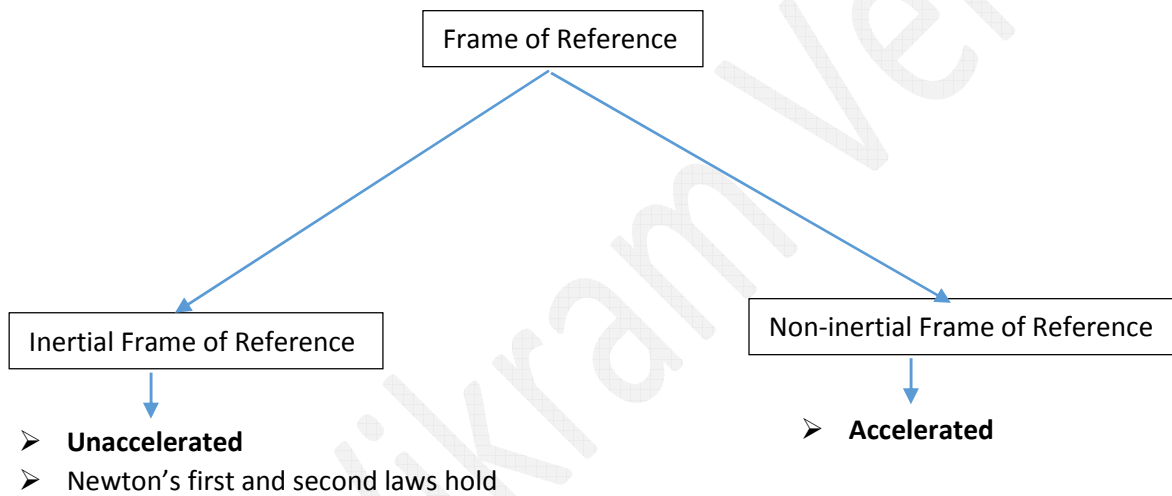


Unit-6: Special Theory of Relativity

Frame of Reference: It is a co-ordinate system attached to a rigid body which describes the position of any particle in space relative to it.

- The simplest frame of reference is Cartesian co-ordinate system having three mutually perpendicular axes X, Y & Z.
- A frame of reference with four co-ordinates (x, y, z, t) is referred to as **Space-Time frame of reference**.

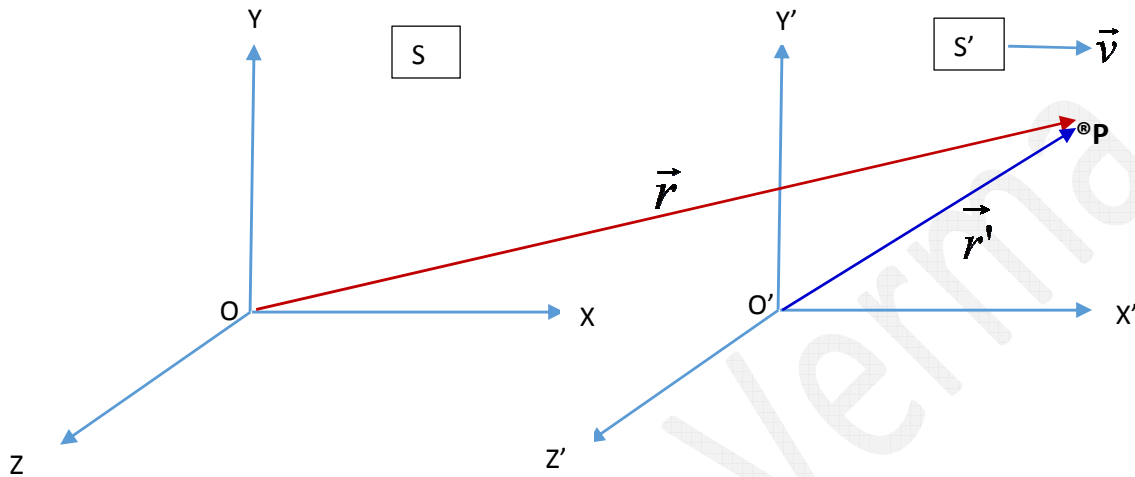


In an inertial frame of reference, if a body is not experiencing any external force, then, its acceleration is given by

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 0,$$

since $(\vec{F} = m \vec{a} = 0)$

- ❖ Let us consider an inertial frame of reference S and another frame S' which is moving with constant velocity \vec{v} relative to S.



Initially ($t = 0$) the origins of both frames coincide (i.e., $OO' = 0$). At any instant t , the position vectors of any particle P is related by

$$\vec{r} = \vec{OO'} + \vec{r}'$$

$$\vec{r} = \vec{v}t + \vec{r}'$$

$$\vec{r}' = \vec{r} - \vec{v}t$$

As velocity \vec{v} is constant, therefore,

$$\frac{d^2\vec{r}'}{dt^2} = \frac{d^2\vec{r}}{dt^2} \implies \vec{a}' = \vec{a}$$

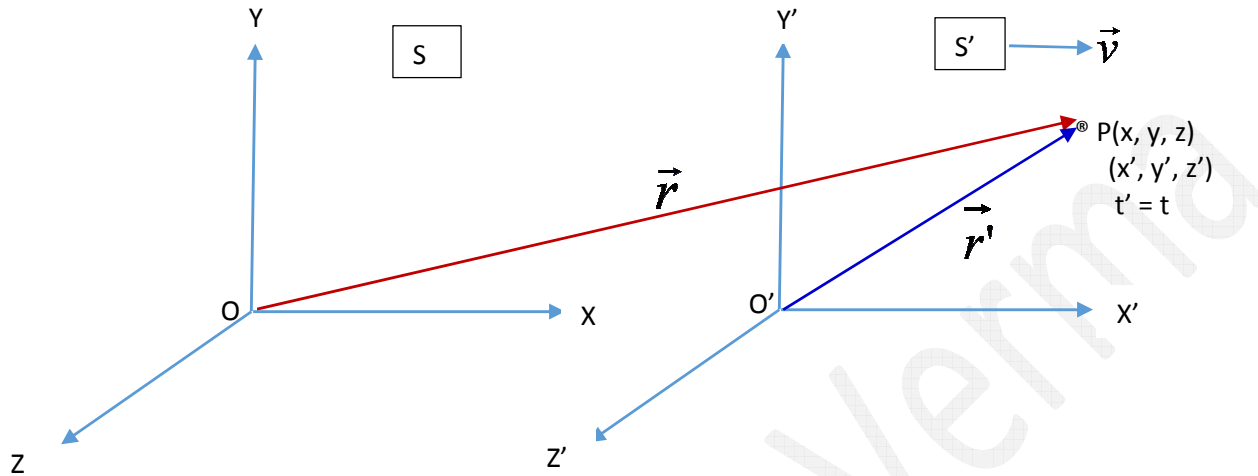
This shows that the particle experiences same acceleration in two frames which are moving with constant velocity relative to each another.

Note-1: If a frame of reference is an inertial frame, then all those frames of reference which are moving with constant velocity relative to the first frame, are also inertial.

Note- 2: The origin of the frame of reference may not always be coincident with the position of the observer; but the frame of reference must, however, be at rest relative to him.

❖ **Galilean Transformation:**

Let us consider two frames of reference S and S'. Assume frame S is inertial and another frame S' is moving with constant velocity \vec{v} relative to S. Hence frame S' is itself an inertial frame.



We assume that-

- i) The origins of two frames coincide at $t = 0$.
- ii) Co-ordinate axes of one frame are parallel to the corresponding co-ordinate axes of another frame.
- iii) Frame S' is moving along X-axis with velocity v relative to frame S.

Then, at any instant t ,

$$\vec{r}' = \vec{r} - \vec{v} t = \vec{r} - v \hat{x} . t \tag{1}$$

In component form,

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned} \tag{2}$$

If it is assumed that time is independent of any particular frame of reference, i.e., if t and t' be the times of occurrence of an event occurring at P, recorded by the observers O and O' respectively, then, $t' = t$.

Thus,

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t \tag{3}$$

These transformation equations are called Galilean Transformation equations.

Inverse Galilean Transformation equations: $x = x' + vt, \quad y = y', \quad z = z', \quad t = t'$ (4)

Differentiating eq.(1) with respect to time t, we have

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}$$

$$\vec{V}' = \vec{V} - \vec{v}$$

(5)

or $\vec{V} = \vec{V}' + \vec{v}$,

Galilean Law of Addition of velocities

where \vec{V}' & \vec{V} are observed velocities in frames S' & S respectively.

→ Velocity is not invariant under Galilean Transformation.

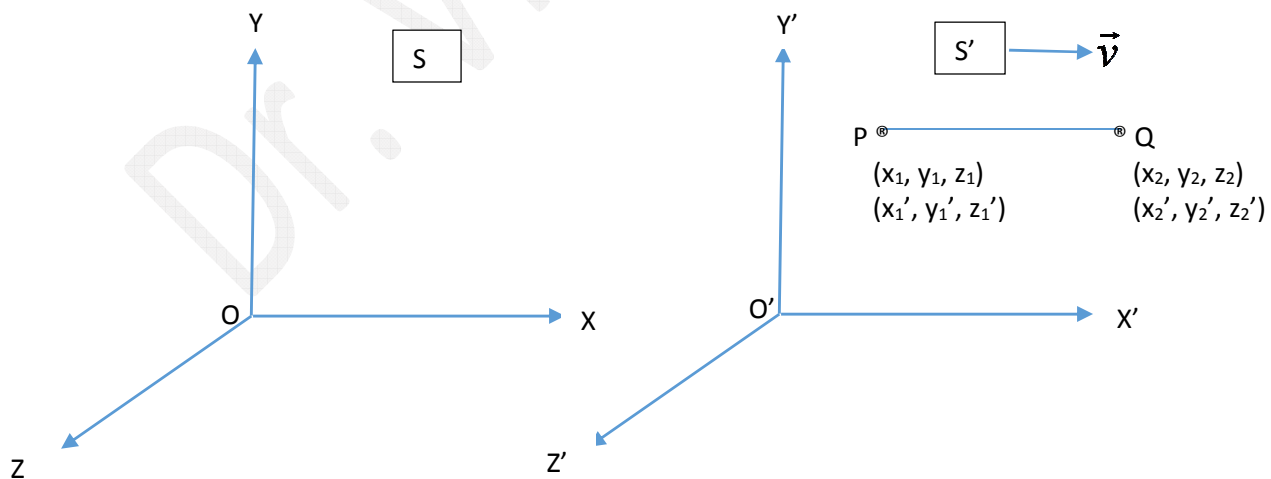
Now differentiating Eq. (5) with respect to time t,

$$\frac{d\vec{V}'}{dt} = \frac{d\vec{V}}{dt} - 0 \quad \rightarrow \quad \vec{a}' = \vec{a}$$

Thus the acceleration of the particle is same in both frames.

→ Acceleration is invariant under Galilean Transformation.

- The length or distance between two points is invariant under Galilean Transformation.



➤ PQ is a rod.

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Class: B.Sc. (H) Semester-II (GE)

Name of Paper: Mechanics

Name of Teacher: Dr. Vikram Verma

In frame S, $L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

In frame S', $L' = \sqrt{(x'_1 - x'_2)^2 + (y'_1 - y'_2)^2 + (z'_1 - z'_2)^2}$

Using Galilean Transformation $\left. \begin{aligned} x'_1 &= x_1 - vt, & y'_1 &= y_1, & z'_1 &= z_1 \\ x'_2 &= x_2 - vt, & y'_2 &= y_2, & z'_2 &= z_2 \end{aligned} \right\}$

$\rightarrow \left. \begin{aligned} x'_1 - x'_2 &= x_1 - x_2 \\ y'_1 - y'_2 &= y_1 - y_2 \\ z'_1 - z'_2 &= z_1 - z_2 \end{aligned} \right\}$

Hence $L' = \sqrt{(x'_1 - x'_2)^2 + (y'_1 - y'_2)^2 + (z'_1 - z'_2)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} = L$

Thus the length or distance between two points is invariant under Galilean Transformation.

References:

1. Mechanics – by Prof. D.S. Mathur; [Page no. 66-105.](#)
2. Mechanics - by J.C. Upadhyaya; [Page no.57-90.](#)
3. An Introduction to Mechanics- by Daniel Kleppner and Robert Kolenkow; [Page no. 341-370.](#)

Note: For further reading and numerical problems, students are advised to read the above books given in reference.

Continued.....