(1) Deductions from Planck's law:

Rayleigh-Jeans law:

Date: / /

We can use Planck's law to obtain all other laws of blackbody radiation as it is the most general description of blackbody radiation. Rayleigh-Jeans law is the special case in the region of longer wavelengths or low frequency. To show this, let us consider

$$\lambda >> \frac{hc}{k_B T}$$

Then the exponential term in equation (17) can be written as -

$$\exp\left(\frac{hc}{\lambda k_B T}\right) = 1 + \frac{hc}{\lambda k_B T} + \text{-----}$$

higher terms can be neglected as $\lambda >> \frac{hc}{k_B T}$.

so that

$$\exp\left(\frac{hc}{\lambda k_B T}\right) - 1 = \frac{hc}{\lambda k_B T}$$

Therefore according to equation (17),

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{\lambda k_B T}{hc}\right) d\lambda = \frac{8\pi k_B T}{\lambda^4} d\lambda$$

which is Rayleigh Jean's law.

(2) Wien's law:

when we consider $\lambda << hc/\lambda k_B T$, the Exponential term in equation (17) become-

$$\exp\left(\frac{hc}{\lambda k_B T}\right) - 1 \;\; \text{\textasciitilde} \;\; \exp\left(\frac{hc}{\lambda k_B T}\right)$$

So $u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \exp\left(\frac{-hc}{\lambda k_B T}\right) d\lambda$

which is wein's law.

(3) wien's displacement law :

Date : / /

As we know λ_m is the wavelength at which the spectral energy density u_λ is maximum. This wavelength can be obtained from equation (17) using the condition –

$$\left[\frac{\partial u_\lambda}{\partial \lambda}\right]_{\lambda=\lambda_{max}} = 0 \qquad \text{——— (18)}$$

Now $$\frac{\partial u_\lambda}{\partial \lambda} = 8\pi hc \frac{\partial}{\partial \lambda}\left[\frac{\lambda^{-5}}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1}\right]$$

$$= 8\pi hc \left[\frac{-5\lambda^{-6}\left[\exp\left(\frac{hc}{\lambda k_B T}\right)-1\right] - \lambda^{-5}\left(\frac{-hc}{\lambda^2 k_B T}\exp\left(\frac{hc}{\lambda k_B T}\right)\right)}{\left(\exp\left(\frac{hc}{\lambda k_B T}\right)-1\right)^2}\right]$$

$$= u_\lambda\left[\frac{-5}{\lambda} + \frac{hc}{\lambda^2 k_B T}\frac{\exp(hc/\lambda k_B T)}{(\exp(hc/\lambda k_B T)-1)}\right]$$

So at $\lambda=\lambda_m$, according to equation (18) –

$$\frac{-5}{\lambda_m} + \frac{hc}{\lambda_m^2 k_B T}\frac{\exp(hc/\lambda_m k_B T)}{(\exp(hc/\lambda_m k_B T)-1)} = 0$$

or $$\frac{hc}{\lambda_m k_B T}\frac{\exp(hc/\lambda_m k_B T)}{[\exp(hc/\lambda_m k_B T)-1]} = 5$$

Let us consider $\frac{hc}{\lambda_m k_B T} = x$, Therefore above equation become –

$$\frac{x\exp(x)}{\exp(x)-1} = 5$$

$\Rightarrow \quad x = 5(1 - \exp(-x))$

The above equation is transcendental equation. This can be solved graphically or numerically.

So we use method of approximation to solve the above equation and guess the value of x - hence the result is -

$$x = \frac{hc}{\lambda_m k_B T} = 4.965$$

or $$\lambda_m T = \frac{hc}{k_B \times 4.965} = b \text{ (constant)}$$

$$\lambda_m T = 2.897 \times 10^{-3} \text{ m-k.} = b$$

This is wien's displacement law.

(4) Stefan's law :

Total energy density (u_λ) for photons of all wavelengths can be obtained by integrating equation (17) for all wavelengths. Therefore

$$u = \int_0^\infty u_\lambda \, d\lambda$$

$$= 8\pi hc \int_0^\infty \frac{d\lambda}{\lambda^5 \left[\exp\left(\frac{hc}{\lambda k_B T}\right) - 1\right]}$$

Let us consider $\frac{hc}{\lambda k_B T} = x \Rightarrow \lambda = \frac{hc}{x k_B T}$

and $d\lambda = -\frac{hc}{x^2 k_B T} dx$.

$$u = 8\pi hc \int_{-\infty}^{0} \frac{\left(\frac{-hc}{x^2 k_B T}\right) dx}{\left(\frac{hc}{x k_B T}\right)^2 \left[\exp(x) - 1\right]}$$

$$= 8\pi k_B^4 T^4 \int_0^\infty \frac{x^3 \, dx}{}$$

The integral $\int_0^\infty \frac{x^3 dx}{\exp(x)-1} = \frac{\pi^4}{15}$

So the total energy density at temperature T is given by –

$$u = \frac{8\pi^5 k_B^4}{15 h^3 c^3} T^4$$

$$u = aT^4$$

where $a = \frac{8\pi^5 k_B^4}{15 h^3 c^3} = 7.56 \times 10^{-16}\ J m^{-3} K^{-4}$

The net rate of flow of radiation per unit area is written as –

$$E = \frac{1}{4} uc = \frac{2\pi^5 k_B^4}{15 h^3 c^2} T^4$$

$$\Rightarrow E = \sigma T^4$$

where σ = stefan Beltzmann constant

$$= 5.67 \times 10^{-8}\ J\ m^{-2} K^{-4} s^{-1}$$

Examples:

1) A body at 1500K emits maximum energy of wavelength 2000 nm. If the Sun emits maximum energy of wavelength 550 nm, what would be the temperature of the Sun?

Sol^n: $T_{body} = 1500K$, $(\lambda_m)_{body} = 2000 \times 10^{-9} m = 2 \times 10^{-6} m$

$T_{Sun} = ?$, $(\lambda_m)_{sun} = 550 \times 10^{-9} m$.

According to wien's displacement law!

$\lambda_m T = \text{Constant}$

So we can write -

$$(\lambda_m)_{body}\, T_{body} = (\lambda_m)_{sun}\, T_{sun}$$

Date: / /

$$\Rightarrow T_{sun} = \frac{(\lambda_m)_{body}\, T_{body}}{(\lambda_m)_{sun}}$$

$$= \frac{2\times10^{-6}\times 1500}{550\times10^{-9}} = 5454\,K$$

So temperature of sun is = 5454 K

2) A black body with temperature 127°c is put in an evacuated enclosure whose walls are blackened and maintained at 27°c. What is net amount of energy lost by body per second per unit area.

Soln:

$$T_{body} = 127^\circ c = 400\,K$$

$$T_{surrounding} = 27^\circ c = 300\,K$$

According to Stefan's radiation law -

$$E = \sigma\left[T_{body}^4 - T_{surrounding}^4\right]$$

$$\sigma = 5.672\times10^{-8}\ J\,m^{-2}K^{-4}s^{-1}\ (Watt\,m^{-2}K^{-4})$$

$$So\ E = 5.672\times10^{-8}\left[(400)^4 - (300)^4\right]$$

$$= 992.6\ watt\,m^{-2}$$

3) Compare the radiant emittance of a black body at 200K and 2000K. $\sigma = 5.672\times10^{-8}\ J\,m^{-2}K^{-4}s^{-1}$

Soln: Here $T_1 = 200\,K$

$$T_2 = 2000\,K$$

at T_1 temp., Stefan's law $E_1 = \sigma T_1^4$

So $\frac{E_1}{E_2} = \frac{T_1^4}{T_2^4} = \left(\frac{T_1}{T_2}\right)^4$

Date: / / /

$\Rightarrow \frac{E_1}{E_2} = \left(\frac{200}{2000}\right)^4 = 10^{-4}$

$\frac{E_1}{E_2} = 10^{-4}$ Ans.

4) A body is kept at 37°c in a room, where temperature is 20°c. How much heat is lost from body in 10 minutes. The surface area of the body is $3\,m^2$ and emissivity is 0.90 and $\sigma = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$.

Soln: Here Area $A = 3\,m^2$

emissivity $e = 0.9$

Body temp. – $T_1 = 37°c = 310K$

Surrounding temp.– $T_2 = 20°c = 293K$

So according to Stefan's law the energy lost per sec is given as –

$E = eA\sigma(T_1^4 - T_2^4)$

$= 0.9 \times 3 \times 5.67 \times 10^{-8}\left[(310)^4 - (293)^4\right]$

$= 286\ J/sec.$

So in 10 minutes (600 sec), energy loss is –

$E \times 10 \times 60 = 286 \times 600$

$= 1.716 \times 10^5\ J$ Ans.