Discipline Course-I

Semester-II

Paper No: Electricity and Magnetism

Lesson: Application of Gauss's law

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Chapter 3A Gauss's law for plane and cylindrical symmetries; and for charged conductors

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- **A.4. Force on the surface of a charged conductor and electrostatic energy in the medium surrounding a charged conductor**

References

Learning Objectives

After going through this chapter, the student would be able to

- Apply Gauss's law to charge distributions with

(a) Planer Symmetry
(b) Cylindrical Symmetry

- Calculate Electric Field due to a charged conductor
- Calculate the electrostatic force and energy on a charged conductor.

A.1 Electric Field due to an infinite sheet of charge and between two parallel charged sheets

Figure 3A.1 shows an infinite plane sheet of charge with constant surface charge density σ. By symmetry, since the sheet is infinite, the field must have the same magnitude and the opposite directions, at two points equidistant from the sheet on opposite sides. To solve the given problem, let us draw a cylinderical Gaussian surface (with one end on one side and other end on the other side) of cross sectional area S. No lines of force cross the cyliderical wall i.e. the component of E normal to the curved surface is zero. Thus the surface integral of $\vec{E}.\vec{dS}$ over the entire surface of cylinder is reduced to the surface integral due to two side surfaces.

$$\int_S \vec{E}.\vec{dS} = \frac{q}{\in_0}$$

$$\text{or} \int_{S_1} EdS + \int_{S_2} EdS = \frac{\sigma S}{\in_0} \Rightarrow 2ES = \frac{\sigma S}{\in_0} \Rightarrow E = \frac{\sigma}{2 \in_0}$$

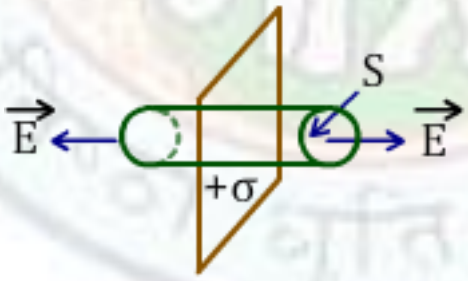


Figure 3A.1 Electric field due to a thin sheet of charge.

Thus we see that the magnitude of the field is independent of the distance from the sheet.

When two plane parallel infinite conducting plates, separated by a distance, are given equal and opposite charges, the field is uniform in the space between them as shown in figure 3A.2. Two infinite parallel planes carry equal but opposite uniform densities ±σ.

We consider three regions to find the field (i) To the left of both the plates (ii) Between them and (iii) To the right of both the plates. The left plate produces a field $\sigma/2\epsilon_0$, which points away from it, to the left in region 1 and to the right in regions 2 and 3. The right plate, being negatively charged, produces a field $\sigma/2\epsilon_0$, which points toward it, to the right, in regions 1 and 2 and to the left in region 3. The two fields cancel in regions 1 and 3; they add up in region 2.

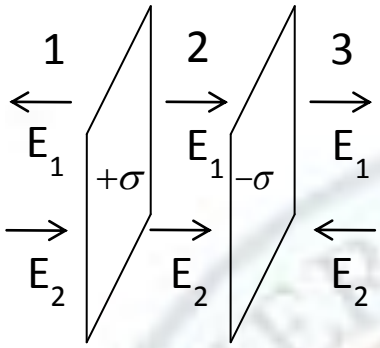


Figure 3A.2 Electric field between the plates of a parallel plate capacitor.

For region 1 and 3

$$\int_S \vec{E}.\vec{dS} = \int_S EdS = ES = \frac{(\sigma S - \sigma S)}{\epsilon_0} = 0 \Rightarrow E = 0$$

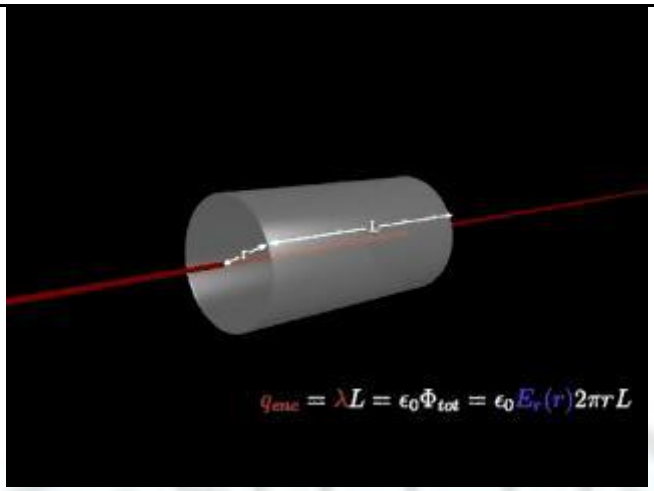
For region 2

$E = E_1 + E_2$

$$\Rightarrow E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Thus the net field is σ/ϵ_0 and points to right, between the plates; elsewhere it is zero.

Value addition: Animation
Electric field due to a Line charge distribution



Draw the animation, stills are provided for your convenience.

Source: http://www.youtube.com/watch?v=0UZwRO8p8Vg&feature=related

A.2 Electric Field due to an infinite line of charge and that due to a charged cylinderical conductor

Let λ be the linear charge density uniformly distributed along an infinite straight line. Let P be a point at a distance r from the line charge where electric field E is required (fig. 4.3). Let us surround a segment of the line charge with a closed cylinderical Gaussian surface, of length l and radius r, co-axial with the line charge, such that point P lies on the cylinderical surface. The cylinder has two types of surfaces (i) Two circular faces S_1 and S_2 and (ii) The curved surface S.

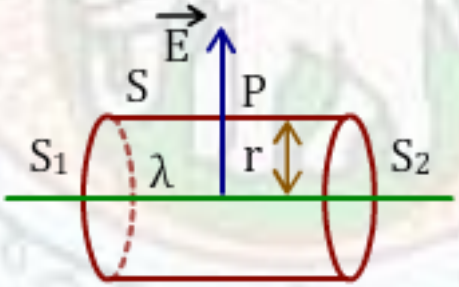


Figure 3A.3 Electric field due to a line charge.

By symmetry, the electric field E is equal in magnitude and directed normally at every point of the curved cylinderical surface. The electric flux, through each circular face, is zero as the angle between the normal to the circular face and the direction of electric field is π/2.

$$\int_{S_1} \vec{E}.\hat{n}\,da = \int_{S_2} \vec{E}.\hat{n}\,da = 0$$

The electric flux due to curved surface $\oint_S \vec{E}.\hat{n}\,da = \oint_S E\,da = E(2\pi rl)$. Thus the total electric flux is due to the curved surface alone.

According to Gauss's law, $E(2\pi rl) = \frac{1}{\in_0} \times$ (charge enclosed by the cylinder)

$$= \frac{1}{\in_0}(\lambda l)$$

$$\Rightarrow E = \frac{\lambda}{2\pi r \in_0} = \frac{1}{4\pi \in_0}\frac{2\lambda}{r}$$

Thus the electric field strength at a distance r, from an infinite linear charge, with linear charge density λ, is directed radially away from the line charge and has a magnitude $\frac{\lambda}{2\pi r \in_0}$ which is inversely proportional to r. The same result will be observed for a point outside a cylindrical conductor (a wire) when the charge is distributed over its surface. The field decreases at points outside.

If the charge is distributed throughout the volume of cylindrical conductor, and its volume charge density is ρ (fig. 4.4), then

$$2\pi rlE = \frac{\rho(\pi R^2 l)}{\in_0}$$

$$\Rightarrow E = \frac{\rho R^2}{2r \in_0}$$

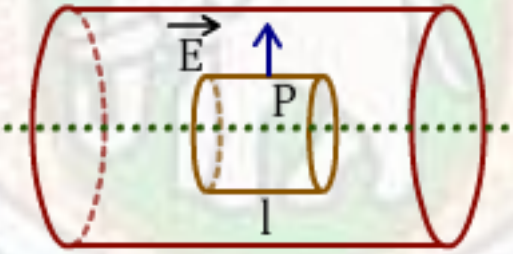


Figure 3A.4 Electric field due to a charged cylinderical conductor (at a point inside the conductor).

The electric field, inside the charged cylinder will be zero if the charge is on its surface only. This is because the net charge, enclosed by the Gaussian surface, through this point, is zero. The case is different when the electric charge is distributed uniformly with in the cylinder of radius R.

Let us now find E at an inner point P, at a distance r, from the axis of the cylinder, of length *l* and of radius r. As the circular faces do not contribute to the flux, the total flux is due to the curved surface for which $\vec{E}$ is perpendicular to the surface. Hence from Gauss's law we have

$$\int_S \vec{E}.\vec{dS} = \int_S EdS = E2\pi rl = \frac{q'}{\in_0}$$

The charge q' inside this Gaussian surface = $\pi r^2 l\rho$

$$E2\pi rl = \frac{\pi r^2 l\rho}{\in_0} \Rightarrow E = \frac{r\rho}{2\in_0}$$

Thus the electric field intensity, at a point inside an infinite uniformly charged cylinder, is radially directed and varies directly as the distance from its axis. Here we see that the electric field, due to a charged cylinder, does not depend upon its radius. Hence we can say that it is same as though the charge on the cylinder were concentrated on a line along its axis

A.3 Electric Field due to a charged conductor

Consider a charged conductor of irregular shape as shown in figure 3A.5. At a small surface δS, let us assume σ to be the surface charge density. Let us construct a Gaussian surface in the form of a cylinder AB of cross section δS, one end face of which is inside the conductor and other passes through a point P outside the surface close to it. The end face B does not contribute to flux as E is zero everywhere inside the conductor.

The curved surface also does not contribute to the flux as $\vec{E}$ is always normal to the charged conductor and hence parallel to the curved surface. Thus the only contribution to the flux is through the end face A which is outside the conductor.

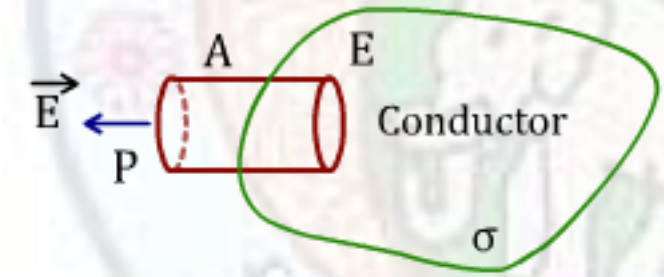


Figure: 3A.5 Electric field due to a charged conductor.

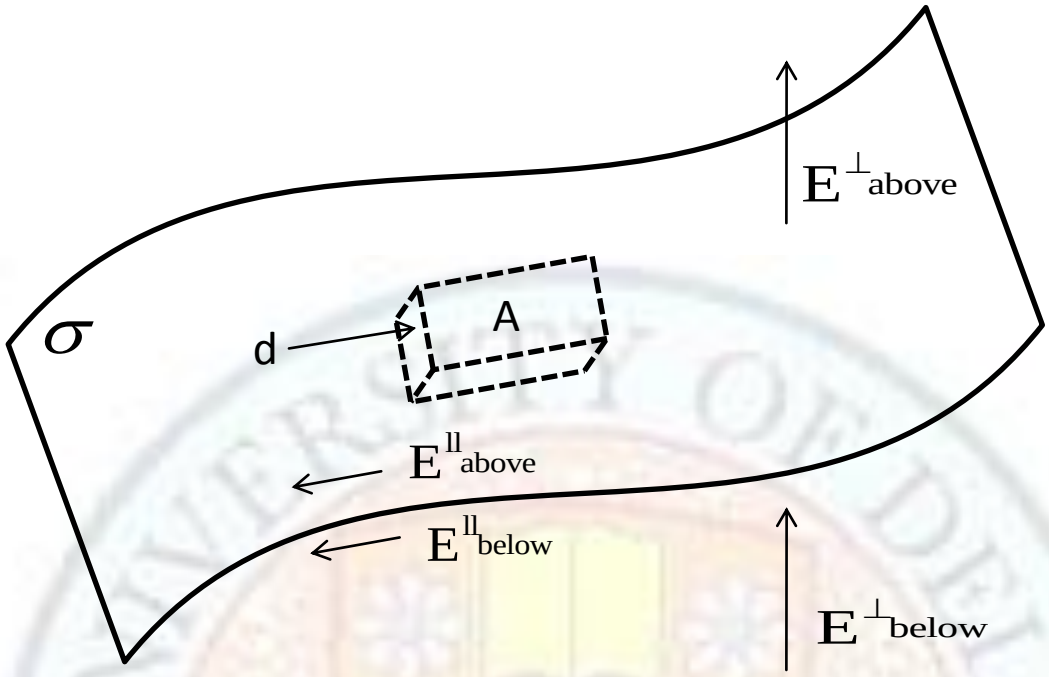
From Gauss's law,

$$\int_S \vec{E}.\vec{dS} = \int_S EdS = E\delta S = \frac{\sigma\delta S}{\in_0} \Rightarrow E = \frac{\sigma}{\in_0}$$

Thus the electric intensity, at any point, close to the charged conductor, is $1/\in_0$ times the surface density of charge on the surface.

Value addition: Did you Know
Electrostatic Boundary Conditions

Body text: The electric field always undergoes a discontinuity when it cross a surface charge σ.



To find the amount by which $\vec{E}$ changes at such a boundary, suppose we draw a wafer-thin Gaussian pillbox. Gauss's law states that

$$\int_S \vec{E}.\vec{dS} = \int_S EdS = \frac{Q_{enclosed}}{\in_0} = \frac{\sigma A}{\in_0}$$

where A is the area of the pillbox. The sides of the pillbox contribute nothing to the flux, in the limit as the thickness d goes to zero, so

$$E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\in_0}$$

where $E^{\perp}_{above}$ denotes the component of $\vec{E}$ that is perpendicular to the surface immediately above and $E^{\perp}_{below}$ is below the surface. Thus the normal component of $\vec{E}$ is discontinuous by an amount $\sigma/\in_0$ at any boundary. Since the thickness is very small (d→0), the tangential component of $\vec{E}$ is always continuous. Thus

$E^{ll}_{above} = E^{ll}_{below}$,

where E^{ll} stands for the components of $\vec{E}$ parallel to the surface. On combining the two equations for perpendicular and parallel components:

$$\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\in_0}\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to the surface, pointing from below to

above.

Reference: David J. Griffiths, Introduction to Electrodynamics, 3rd edition.

A.4 Force on the surface of a charged conductor and Electrostatic Energy in the medium surrounding a charged conductor

We know that the field inside a conductor is zero, and, from the boundary conditions, the field outside is

$$\vec{E} = \frac{\sigma}{\in_0} \hat{n}$$

where σ denotes the charge distribution over the surface of conductor.

In the presence of an electric field, a surface charge will experience a force and the force per unit area $\vec{F} = \sigma \vec{E}$. But the electric field is discontinuous at a surface charge, so we use the average of two: $E^{\perp}_{above}$ and $E^{\perp}_{below}$

$$\vec{F} = \sigma \vec{E}_{average} = \frac{1}{2}\sigma(\vec{E}_{above} + \vec{E}_{below})$$

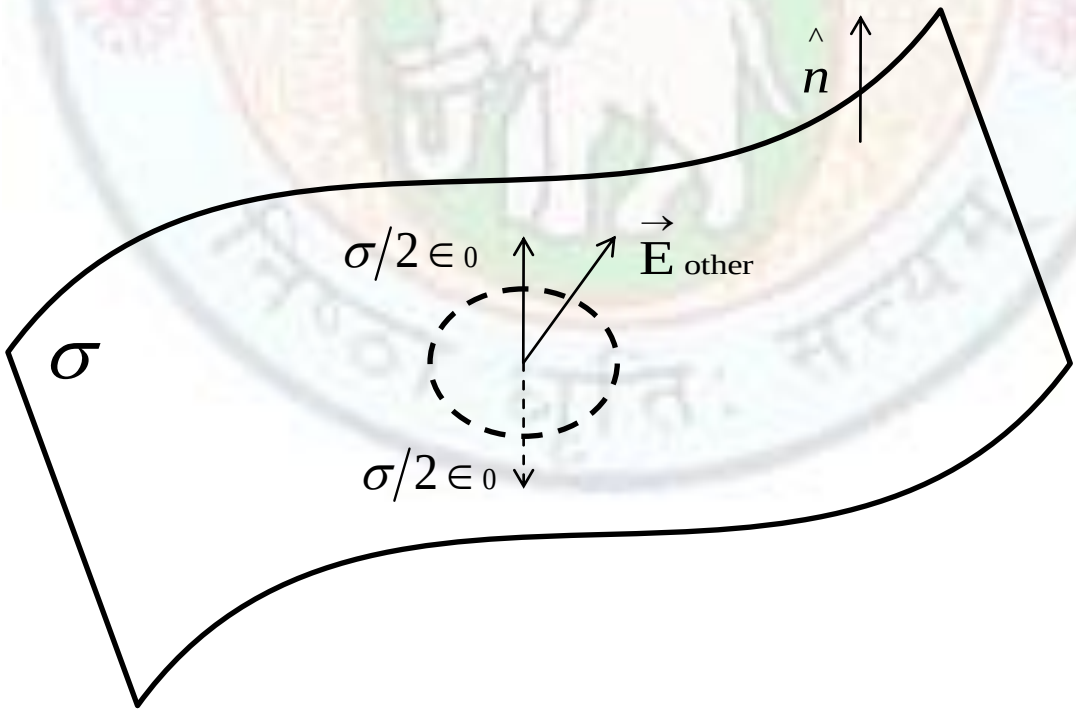


Figure: 3A.6 Electric field due to a charged conductor.

Let us take a very small part of surface in figure 3A.6. The total field consists of two parts: that due to the tiny part itself and that due to the other regions of the surface.

$$\vec{E} = \vec{E}_{\text{tiny part}} + \vec{E}_{\text{other}}$$

The force on the tiny part is only due to $\vec{E}_{\text{other}}$, because it cannot exert a force on itself. The discontinuity is only due to the charge on the patch, which puts out a field $\sigma/2\in_0$ on either side, pointing away from the surface. Thus,

$$\vec{E}_{\text{above}} = \vec{E}_{\text{other}} + \frac{\sigma}{2\in_0}\hat{n}$$

$$\vec{E}_{\text{below}} = \vec{E}_{\text{other}} - \frac{\sigma}{2\in_0}\hat{n}$$

Hence $\vec{E}_{\text{other}} = \vec{E}_{\text{average}} = \frac{1}{2}(\vec{E}_{\text{above}} + \vec{E}_{\text{below}})$. This relation is true for any surface charge. This is because the field inside a conductor is zero and that outside is $(\sigma/\in_0)\hat{n}$. Hence the average is $(\sigma/2\in_0)\hat{n}$ and the force per unit area is

$$\vec{F} = \frac{\sigma^2}{2\in_0}\hat{n}$$

This amounts to an outward electrostatic pressure on the surface, tending to draw the conductor into the field. This pressure is given by:

$$P = \frac{\sigma^2}{2\in_0} = \frac{1}{2}\in_0 E^2 = \text{ energy per unit volume}$$

Hence we may say that energy gets stored in the field with an energy density $\frac{1}{2}\in_0 E^2$.

Value addition: Did you Know
Lightning-What is it? How it is formed?
Body text:



The above photograph is the snapshot of the video (for identifying the video)

Source:

Question Number	Type of question
1	MCQ

Question

(1) A charge distribution produces the field $\mathbf{E} = r^2\,\mathbf{r} - r^2 \sin\theta\,\boldsymbol{\theta} + 10\,\boldsymbol{\phi}$. Determine the charge density (C/m^3) at (2, π, 0).

Option(s)	
	(a) $2\,\epsilon_o$
	(b) $10\,\epsilon_o$
	(c) $12\,\epsilon_o$
	(d) None

Question

(2) An electric field is given by **E** = xy^2 **x** – xz^2 **y** + *xyz* **z**. Calculate the x component of its curl at (2, 1, 3).

Option(s)	(a) 18 (b) 10 (c) 8 (d) None

Correct Answers	1) c 2) a

Question Number	Type of question
2	Subjective

Question

A cylindrical surface of radius λ that is infinite in length and supports a uniform surface charge distribution ρ_s (C/m^2). Determine the potential difference between two points at radial distances r_a and r_b from the axis, with $r_b > r_a > \lambda$.

Correct Answers	Using symmetry and Gauss's law, the **E**-field is directed radially away from the cylinder and is given by E = (ρ_s λ/ ϵ_or), for points outside the cylinder. Then V_b-V_a = - $\int_{ra}^{rb} E.dr = \frac{\rho\lambda}{\epsilon o} ln \frac{ra}{rb}$

References:

1. **Introduction to Electrodynamics, 3rd edition, David J. Griffiths.**
2. **Electricity and magnetism. By D C Tayal (Himalaya Publishing House,1988).**
3. **Electricity and magnetism By Edward M. Purcell (McGraw-Hill Education, 1986).**

4. **University Physics, 6th edition, F. W. Sears, M. W. Zemansky and H. D. Young (Narosa Publishing House, 1990).**
5. **Problems in General Physics, I. E. Irodov (Mir Publishers Moscow, 1988).**

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Learning Objectives

After going through this chapter, the student would be able to

- Use Gauss's for deriving Coulomb's Law
- Apply Gauss's law to charge distributions with spherical symmetry

B.1 Applications of Gauss's Law

Gauss's law is useful incalculating the electric field in problems, in which it is possible to choose a closed surface such that the electric field has a normal component which is either zero or has a single fixed (constant) value at every point on the surface. Such a closed surface, for a given charge distribution, is referred to as the 'Gaussian Surface' for that charge distribution. We can normally find such a Gaussian surface only for charge distributions that have a high degree of symmetry associated with them. It is only when we are able to think of a suitable Gaussian surface (for a given charge distribution) that we are able to use Gauss's theorem for a simple calculation of electric fields.

B.1.1 Electric Field due to a point charge (Coulomb's Law)

We have considered Coulomb's law as fundamental equation of electrostatics and have derived Gauss's law from it. However Coulomb's law can also be derived from Gauss's law. This is done by using this law to obtain the expression for the electric field due to a point charge.

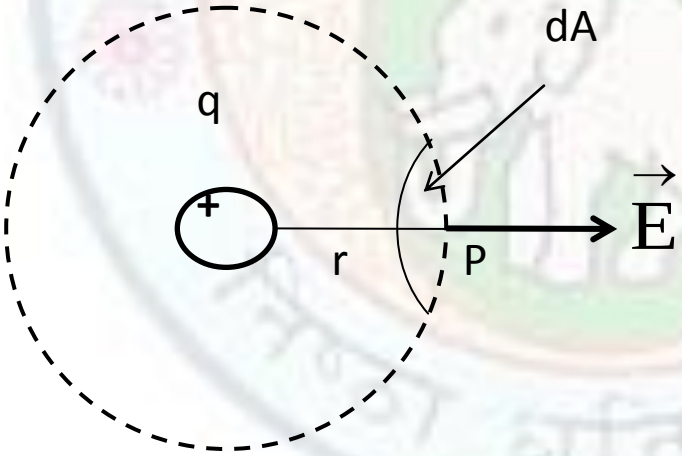


Figure 3B.1 Electric field at a point on the spherical Gaussian surface surrounding a point charge.

Consider the electric field due to a single positive point charge q. By symmetry, the field is everywhere radial and its magnitude is the same at all points, that are at the same distance r from the charge as shown in figure 3B.1. Hence, if we select, as a Gaussian surface, a spherical surface of radius r, $E_{\perp} = E = \text{constant}$ at all points on this surface and the field is radial. If we consider a small elementary area of this gaussian surface, the area vector is in the radial direction i.e. perpendicular to surface. Then

$$\int_S \vec{E}_{\perp} . d\vec{A} = \int_S E_{\perp} \, dA = E_{\perp} \int_S dA = EA = E(4\pi r^2)$$

From Gauss's law

$$E(4\pi r^2) = \frac{q}{\in_0}$$

$$\Rightarrow E = \frac{1}{4\pi \in_0}\frac{q}{r^2}$$

The force on a point charge q' at a distance r from the charge q is then

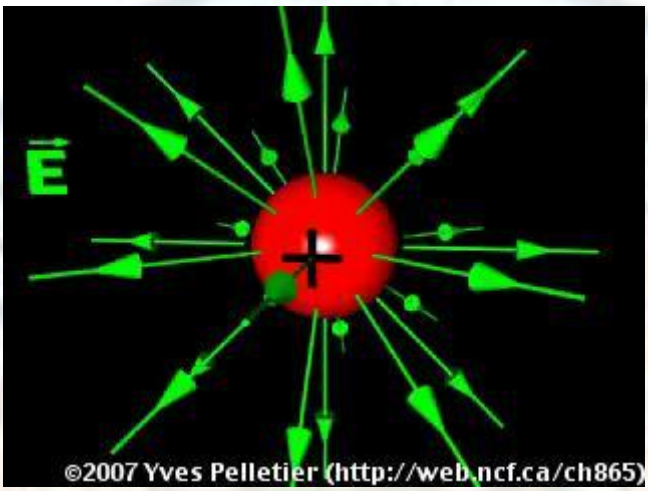
$$F = q' E = \frac{1}{4\pi \in_0}\frac{qq'}{r^2}$$

which is Coulomb's law.

Value addition: Did you Know
Gauss's law and inverse square laws.
Body text: Gauss's law is a direct consequence of the inverse square nature of Coulomb's law. The two laws can be viewed as two different ways of expressing the same basic fact-the inverse square nature of the force between two point charges. This also implies that we can think of a 'Gauss's law' for any force field (like the Gravitational force) that has an 'inverse square' nature.
Reference: Electricity and magnetism By Edward M. Purcell (McGraw-Hill Education, 1986).

B.1.2. Electric Field due to a uniformly charged sphere

A spherically symmetric distribution of charge means the distribution of charge where the charge density ρ depends only on the distance of the point from the center and not on the direction. Let the spherically symmetric charge distribution be characterised by a charge density function, $\rho(r)$, which varies in a certain manner, with distance, from the center of the spherical surface. Consider first the case of a charge q that is uniformly distributed over a sphere of radius R. Let us calculate the electric field strength at any point distant r from the centre.

Value addition: Did you Know
Electric field of a charged sphere
Body text: 
Source: http://www.youtube.com/watch?v=A_NcMQA5RQc&feature=BF&list=PLF9950627F5C4ECA2&index=14

(i) Electric field strength at an external point

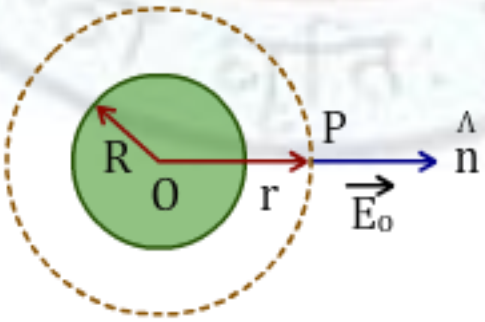


Figure 3B.2 Electric field at a point on the spherical Gaussian surface surrounding uniformly charged sphere.

Let P be a point outside the charge distribution of radius R and centre O. Let OP=r. Consider a sphere of radius OP=r concentric with the spherical charge. As the distribution of the charge is uniform, by symmetry, the electric field strength E_0 at every point of this spherical surface has the same magnitude and is directed along the outward normal to the surface as shown in figure 3B.2.

The net surface integral, of the normal component of the electric field, i.e. net electric flux through the whole surface, $\int_S \vec{E}_0.\vec{da} = \int_S E_0 da = E_0 \int_S da = E_0(4\pi r^2)$. But according to Gauss's law, total electric flux must be equal to $\frac{1}{\in_0}$ times the total charge enclosed by the surface i.e.

$$E_0(4\pi r^2) = \frac{1}{\in_0}\int_V \rho dV = \frac{1}{\in_0}\int_0^R \rho 4\pi x^2 dx$$

$$E_0 = \frac{1}{4\pi \in_0}\frac{1}{r^2}\int_0^R \rho(x) 4\pi x^2 dx$$

Here $\rho(x)4\pi x^2 dx$ represents the charge on the shell of radius x and thickness dx, so that $\int_0^R \rho(x)4\pi x^2 dx$ is the total charge enclosed by the surface $= q$.

$$\therefore\ E_0 = \frac{1}{4\pi \in_0}\frac{q}{r^2}$$

which is the same if the charge q were placed at the centre O.

Hence the electric field strength, at any point outside a spherical charge distribution, is the same as through the whole charge were concentrated at the centre.

(ii) Electric field strength at the surface of the spherical charge distribution

In this case, the point P lies on the surface of the spherical charge i.e. r=R. Hence the electric field strength, on the surface of the spherical charge distribution, is

$$E_S = \frac{1}{4\pi \in_0}\frac{q}{R^2}$$

(iii) Electric field strength at an internal point

Let P be an internal point at a distance r (r < R)from the centre of the charge distribution. Consider a sphere of radius (OP=r) concentric with the spherical charge (figure 3B.3). Let ρ be the volume charge density (charge per unit volume)

$$\therefore\ \rho = \frac{\text{charge}}{\text{volume}} = \frac{q}{\frac{4}{3}\pi R^3}$$

Let the whole surface be divided into thin spherical shells. The electric field strength E_i, at P, is the combined effect of shells outside the spherical surface of radius r, as well as

those inside it. But the electric field strength contribution due to the outer spherical shells, is zero which may be seen as follows:

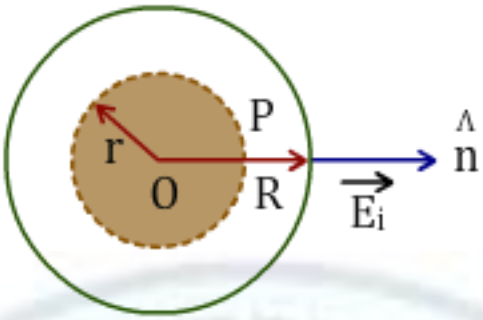


Figure 3B.3 Electric field at a point inside uniformly charged sphere.

Consider any point P, inside a charged thin shell of radius x. We have to find the electric field strength at P due to this shell. Consider a spherical surface of radius OP (= r) concentric with this spherical shell. By symmetry, the electric field strength, $\vec{E}$, at every point of this spherical surface, has the same magnitude and is directed along the outward drawn-normal to this surface. The electric flux through the whole surface $\int_S \vec{E}.\vec{da} = \int_S E da = E\int_S da = E(4\pi r^2)$. According to Gauss's theorem, $E4\pi r^2 = \frac{1}{\in_0} \times$ total charge enclosed by the surface. This equals zero since the net charge enclosed by this internal surface is zero. This implies that E=0.

Thus electric field strength, due to a charged spherical shell, at an internal point is zero. Hence the electric field strength, E_i, at P, is due to the inner shells only, which may be found as follows:

By symmetry, the electric field strength E_i at every point of the spherical surface of radius r has the same magnitude and is directed along the outward drawn normal to the surface. The total electric flux through the whole surface $\int_S \vec{E}_i.\vec{da} = \int_S E_i da = Ei\int_S da = E_i(4\pi r^2)$.

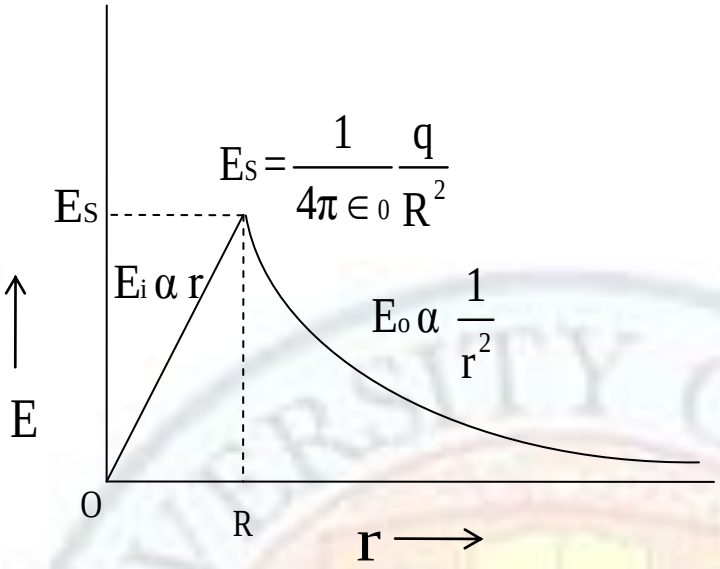


Figure 3B.4 Electric field due to a uniformly charged sbhere as a function of distance from the centre of sphere.

According to Gauss's law, $E_i 4\pi r^2 = \frac{1}{\in_0} \times$ charge enclosed by the Gaussian surface

$$= \frac{1}{\in_0} \int_0^r \rho(4\pi x^2)dx = \frac{1}{\in_0} \rho \frac{4}{3}\pi r^3$$

$$\Rightarrow E_i = \frac{1}{4\pi \in_0}\left(\frac{4}{3}\pi r\rho\right) = \frac{1}{4\pi \in_0}\frac{4}{3}\pi r.\frac{q}{\frac{4}{3}\pi R^3} \qquad \left(\because \text{ since } \rho = \frac{q}{\frac{4}{3}\pi R^3}\right)$$

$$= \frac{1}{4\pi \in_0}\frac{qr}{R^3}$$

Thus the electric field strength, at a point P inside a spherically symmetric charge distribution, is directly proportional to the distance of the point P from the centre of the spherical charge. The variation, of the magnitude of the electric field strength, with distance, from the centre of a spherically symmetric charge distribution, is, therefore, represented by the curve shown in figure 3B.4.

B.2 Electric Field due to a charged spherical shell or charged conducting sphere

In the case of a charged spherical conductor, the charge is distributed uniformly over its outer surface and there is no charge inside it. The field at any external point has the

same symmetry as that of a point charge. We can construct a gaussian surface of radius r>R. The electric intensity is given by $E = \frac{1}{4\pi \in_0} \frac{q}{r^2}$

At the surface r=R we have $E = \frac{1}{4\pi \in_0} \frac{q}{R^2}$.

At all points inside a charged spherical conductor, or hollow spherical shell, electric field E=0 as there is no charge inside such a sphere.

Value addition: Animation

Example 1

Body text:

Story board:

Step1: In a thunderstorm accompanied by lightening, a person standing under a tree in the open ground, then electricity (due to lightening) discharges through the body of that person to the ground.

Step2: In a thunderstrom accompained by lightening, it is safer to be inside a car than to be in the open ground. The metallic body of the car provides electrostatic shielding from the lightening. This is a consequence of the fact that the electric field, inside a charged conductor, is zero.



Reference: David J. Griffiths, Introduction to Electrodynamics, 3rd edition.

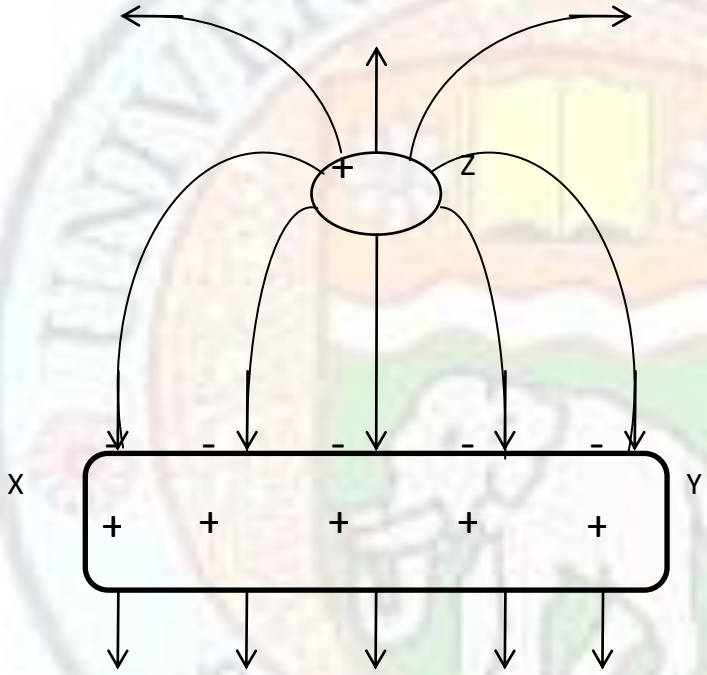
Value addition: Interesting facts

Electric Shielding

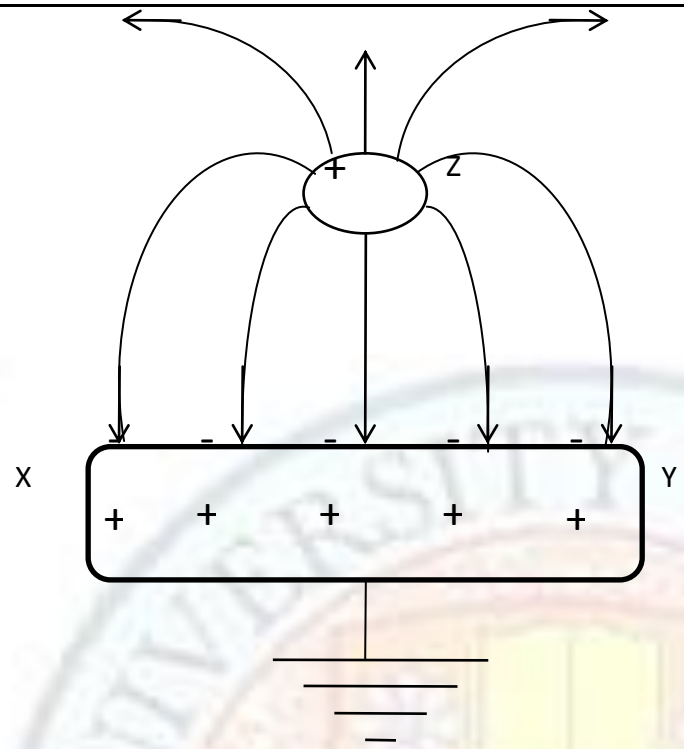
Body text:

Electrostatic sheiding/screening is the phenomenon of protecting a certain region of space from unwanted effects of external electric fields.

We know electric field inside a conductor is zero. Therefore, to protect an instruments from external electric fields, we enclose them in hollow conductors. In a thunderstrom accompained by lightening, it is safer to be inside a car than to be in the open ground. The metallic body of the car provides electrostatic shielding from the lightening. Further, an earthed conductor XY can also act as a screen against the electric field.



When XY is not earthed, field of the charged body Z, due to electrostatic induction, continues beyond XY. However when XY is earthed, the induced positive charge flows to earth and the field in the region beyond XY disappears. This prevents the electrostatic field of the generator from spreading out of its enclosure or cage.



Reference: David J. Griffiths, Introduction to Electrodynamics, 3rd edition.

Value addition: Did you Know
Science Explains-Lightning
Body text: The below photograph is the snapshot of the video (for identifying the video)



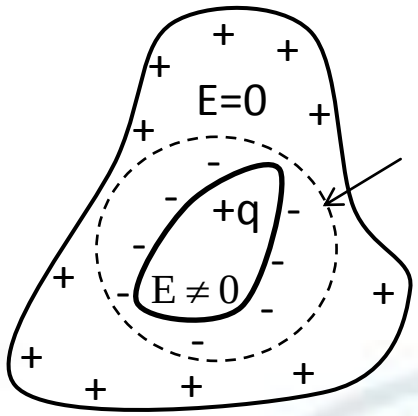
Source: http://www.youtube.com/watch?v=ipAemhMEKwE

Value addition: Did you Know

Induced charges

Body text:

If there is some cavity in the conductor and within that cavity there is some charge, then the field in the cavity will not be zero. But the cavity and its contents are electrically isolated from the outside world by the surrounding conductor. No external fields penetrate the conductor; they get cancelled at the outer surface by the induced charge there. Similarly, the field due to charges within the cavity is cancelled off, for all exterior points, by the induced charge on the inner surface. But the outer surface of the conductor, having an induced charge q, interacts with the outside world. The total charge induced on the cavity wall, is equal and opposite to the charge inside. This is because if we surround the cavity with a Gaussian surface, all points of which are in the conductor, the net enclosed charge must be zero, $Q_{enclosed}=q+q_{induced}=0 \Rightarrow q_{induced} = -q$. Using Gauss's law, we get $\int_S \vec{E}.\vec{da} = 0$.



Reference: David J. Griffiths, Introduction to Electrodynamics, 3rd edition.

Question Number	Type of question
1	Subjective

Question

A conducting spherical shell, of inner radius a, and outer radius b, has a positive point charge q located at its center. The total charge on the shell is zero and it is insulated from its surroundings. Find the electric field magnitude in terms of the distance r from the center for the regions (i) r<a (ii) a<r<b and (iii) r>b.

Solution

(i) For the region r<a, the electric field is only due to the point charge (q) at its center. Hence $E = \frac{1}{4\pi \in_0} \frac{q}{r^2}$

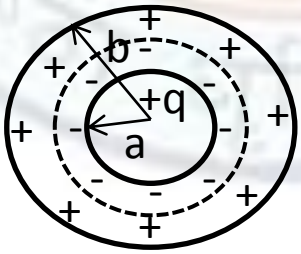


Figure 3B.5 A conducting spherical shell.

(ii) For the region (a<r<b), the total charge induced on the wall of inner shell is equal and opposite to charge inside. We surround the inner shell with a Gaussian surface, all points of which are in

	the conductor. Hence, $\int_S \vec{E}.\vec{da} = 0$. By Gauss's law, this implies that the net enclosed charge must be zero i.e. $Q_{enclosed} = q+q_{induced} = 0$. Hence $q_{induced} = -q$. For the region (r>b), the electric field is due to the induced charge on the outer shell. Hence $E = \frac{1}{4\pi \in_0} \frac{q}{r^2}$.

Question Number	Type of question
2	Subjective

The passage of a thundercloud, overhead, caused the vertical electric field strength in the atmosphere, measured at the ground, to rise to 3000 V/m.

(a) How much charge did the thundercloud contain, in coulomb per m^2 of the horizontal area?

(b) Suppose there was enough water in the thundercloud in the form of 1mm diameter drops to make 0.25 X 10^{-2} m of rainfall, and that it was those drops which carried the charge. How large was the electric field strength at the surface of one of the drops?

Solution

(a) We have a relation between charge per unit area and the field strength

$\sigma = E\epsilon_0$

$\therefore \sigma = 3000 \times 8.85 \times 10^{-12}$ C/m^2

$\sigma = 2.655 \times 10^{-8}$ C/m^2

(b) Volume of a single drop = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi$ mm^3 = $\frac{4}{3}\pi \times 10^{-9}$ m^3

Total volume (to make 0.25 X 10^{-2} m of rainfall) V= 1 × 1 × 0.25 × 10^{-2} m^3 = 0.25 × 10^{-2} m^3

And number of drops in this volume = N = $\frac{V}{\frac{4}{3}\pi R^3}$ = 6 × 10^5 drops

The electric field strength at the surface of one of the drops = E/N

	$=(3000 \times 10^{-5}/6)$ V/m $= 500 \times 10^{-5}$ V/m $= 0.5 \times 10^{-2}$ V/m.

References

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